

# Post-Modern Portfolio Theory and Municipal Revenue: Modeling Revenue Structures with Risk Asymmetry\*

Nathan J. Grasse  
Carlton University

Wesley R. Janson  
University of Chicago

Elizabeth A.M. Searing  
University of Texas at Dallas

Justin Marlowe  
University of Chicago

## Abstract

Revenue diversification is relevant to multiple fields of public administration, including public choice theory and financial management. Municipal governments rely on revenue management to fund service provisions, and conventional wisdom suggests that diversification is beneficial. Although the Hirschman-Herfindahl Index (HHI) is commonly used to measure municipal revenue diversification (concentration), it overlooks key characteristics such as revenue growth, risk, and covariance that underpin the rationale for diversification based on modern portfolio theory (MPT) (Markowitz, 1952; Grasse et al., 2016; Janson and Marlowe, 2023). Scholars have applied MPT to state government, municipal, and nonprofit revenue portfolios (White, 1979; White and Chou, 1980; Berg et al., 2000; Harmon and Mallick, 1994; Grasse et al., 2016); this paper extends that work by applying post-modern portfolio theory (PMPT) to financial data from Minnesota municipalities (2003–2022). We identify optimal portfolios through mean-variance and mean-semivariance analysis, offering insights into the importance of risk measurement—particularly risk asymmetry—and how municipal service profiles can shape efficient revenue structures. This demonstrates the importance of risk definition, which affects portfolio composition, return, and the theoretically ideal level of diversification. Our results have wide-ranging implications from local government administration to evaluation in municipal bond markets.

**Keywords:** Portfolio Theory, Municipal Revenue Management, Asymmetric Risk, Minnesota Municipalities

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\*Grasse: Nathan.Grasse@carleton.ca; Janson: wr.janson@gmail.com; Marlowe: Center for Municipal Finance, Harris School of Public Policy, jmarlowe@uchicago.edu; Searing: Elizabeth.Searing@utdallas.edu. We graciously thank, without implicating, John Jernberg at the Minnesota Office of the State Auditor for his assistance in obtaining and deciphering the data. We also sincerely thank Janosch Ortmann (Université du Québec à Montréal), who greatly improved the paper by reviewing our mathematical notation.

# 1 Introduction

Municipal governments provide services and require funds to support these activities. The conventional wisdom in managing this revenue holds that diversification is beneficial, whereas the public choice literature posits that increasingly complex revenue structures may reduce accountability. The first notion is based on modern portfolio theory (MPT), originally developed to identify the optimal asset allocation given estimates of their expected returns, risks, and covariances (Markowitz, 1952). The second notion is based on fiscal illusions that arise from both the complexity of the revenue structure and the nature of specific revenue sources.

Regardless of whether research focuses on financial health or accountability, the most commonly used measure of municipal revenue structures is the Hirschman-Herfindahl Index (HHI); however, the HHI only captures revenue portfolio diversity (or concentration) and omits other critical information about the composition of revenue streams it represents. This means that HHI alone will likely be poorly suited to understanding revenue structures or their implications, since it does not provide specific information on the revenue sources incorporated into the revenue structure (such as source growth, risk, and covariance) that conceptually underlie the logic of diversification (Janson and Marlowe, 2023) in the literature on financial management (Grasse et al., 2016). Furthermore, HHI does not measure revenue sources' contributions to the portfolio, which facilitates the association of these sources with factors that might lead to fiscal illusions. In this way, the literature that relies on diversification has based hypotheses on potentially flawed logic, irrational expectations about monotonic associations, and a limited measure; respectively, diversification is beneficial regardless of revenue structure composition, that changing it will consistently lead to more or less of an outcome, and that the specific composition of a revenue structure is not critical to its qualities. Given this, the inconsistent results of studies which rely on HHI are unsurprising.

A smaller literature directly applies MPT government revenue portfolios (White, 1979; White and Chou, 1980), including municipal portfolios (Berg et al., 2000; Harmon and Mallick, 1994; White and Chou, 1980). This work has relied on symmetric risk measures and mean-variance optimization. This method is undoubtedly an improvement over measures of diversification alone, but we argue that, while symmetric measures of risk may be appropriate in some cases, failing to carefully consider the nature of revenue risk could lead scholarship to generate misleading findings about revenue structures and diversification. At the same time, using these measures could also lead municipal governments to pursue flawed revenue strategies. We therefore introduce post-modern portfolio theory (PMPT) to the literature on municipal revenue management and consider its potential to advance the fields of public financial management and public finance.

This paper focuses on financial management, specifically revenue management, applying portfolio theory to the revenue structures of Minnesota’s municipal governments. Using a panel of financial data from 2003 to 2022, it identifies theoretically optimal portfolios with MPT’s mean-variance and PMPT’s mean-semivariance techniques. It informs the study of municipal revenue structures in two ways: 1) by assessing the importance of risk measurement in portfolio analysis, introducing risk asymmetry to distinguish between upside and downside deviations, and 2) by providing information on the importance of context to revenue management by identifying how municipal characteristics could condition the specification of portfolio efficiency.

The paper also has broader implications for future scholarship, demonstrating that more nuanced measures of revenue structures and risk are required to understand their associations with outcomes relevant to financial management or public choice theories. It therefore concludes with an agenda for using such techniques to study financial health and tax effort or burden.

## 2 Literature Review

### 2.1 Public Sector Revenue Structures

As noted by [Jimenez and Afonso \(2022\)](#), in the public sector, diversification, measured by HHI, has been linked to financial management theory, particularly revenue management, and to public choice theory, specifically fiscal illusion ([Suyderhoud, 1994](#)). This includes empirically testing associations between diversification and revenue capture ([Chernick et al., 2011](#)), revenue volatility ([Yan, 2012](#)), tax volatility ([Afonso, 2013](#)), financial stability ([Carroll, 2009](#); [Shoag et al., 2019](#)), reduced fiscal stress ([Misiolek and Elder, 1988](#)), financial performance ([Suyderhoud, 1994](#)), tax effort ([Hendrick, 2002](#)), spending ([Park and Park, 2018](#)) and spending volatility ([Hendrick and Crawford, 2014](#); [Afonso, 2017](#)). Despite a wide-ranging exploration, studies have yielded mixed results, for example, in identifying positive associations between diversification and both volatility and stability, leading to considerable uncertainty in the literature. As noted above, this conundrum suggests that the underlying relationship may be misspecified in models relying solely on diversification measures, may be contextual, or may be more complex than can be captured by a measure such as HHI.

Although MPT approaches have many advantages relative to HHI, direct applications of modern portfolio theory in the public sector are uncommon. Much of this has focused on state revenue structures. Early work by [White and Chou \(1980\)](#) identified efficient state-level revenue portfolios and frontiers, suggesting optimal tax structures to minimize instability. [Perdue and Weed \(1991\)](#) also suggests that the portfolio

method can provide states with a way to find “the minimum variance tax structure that can meet the fiscal needs of the jurisdiction” (367). [Harmon and Mallick \(1994\)](#) also applies theory to New York State revenues, including equity as a concern and finding similarity between efficient and actual structures, and also identifies tradeoffs between equity and stability. [Garrett \(2009\)](#) uses a sample of state tax revenue data to assess the actual tax portfolios of states relative to those identified using MPT, identifying choices that are likely to introduce unnecessary risk; recommending the application of a portfolio approach in addition to more traditional single-dimensional parametric methods due to the advantages conveyed by the identification of theoretically ideal portfolios.

Research on revenue portfolios has also focused on local government portfolios. White and Chou’s early work identified efficient portfolios among municipalities in the US state of Georgia. Local government studies have also incorporated additional measures, such as inherent equity in revenue streams, into the optimization function ([Mallick and Harmon, 1994](#); [Berg et al., 2000](#)). Symmetric portfolio models have also been applied to the revenue structures of international cities, such as Tehran ([Gholizadeh and Aminirad, 2018](#)). At this point, PMPT has not been applied to government financial management at any level.

## **2.2 Nonprofit Sector Revenue Structures**

We also note the nonprofit literature on revenue portfolios, as it also relies heavily on measures of diversification and includes applications of portfolio theory. The structure of nonprofit revenue portfolios draws from the numerous literatures with varying rationales and implications for diversification. Much of the nonprofit finance literature has roots in resource dependency theory, which holds that organizations dependent on a narrow range of funding sources become vulnerable to external pressures that can threaten their autonomy and long-term sustainability. [Froelich \(1999\)](#) examined the effects of three major revenue strategies in nonprofit organizations (private contributions, government funding, and commercial activities) and demonstrated how evolving resource dependence raises critical questions about nonprofit performance, legitimacy, and public policy. The theory predicts that by spreading revenue across multiple streams, nonprofits can buffer themselves against the loss of any single funding source (for example [Kim \(2024\)](#)). [Chang and Tuckman \(1994\)](#) were among the first scholars to empirically examine revenue diversification and its relationship to financial vulnerability, including an HHI as one of four metrics designed to predict hardship.

Modern Portfolio Theory (MPT) draws on the same base concerns about mitigating risk, though with more complexity than resource dependency theory. Early in the study of nonprofit revenue portfolios, [Kingma \(1993\)](#) and [Jegers \(1997\)](#) both suggested that portfolio theory can be applied to nonprofit revenue

structures. Consequently, there have been direct applications of portfolio theory in the nonprofit literature, although they are based on symmetrical approaches (Grasse et al., 2016; Qu, 2019), as well as studies exploring diversification and the influence of individual revenue streams on revenue capture and volatility to better identify the impact of diversification (Mayer et al., 2014).

There is a tension between risk reduction and increased costs. For example, Carroll and Stater (2009) find that nonprofits can reduce revenue volatility through diversification, particularly by equalizing reliance on earned income, investments, and contributions. However, the authors acknowledged that generating revenue from multiple sources can burden nonprofit organizations since managing multiple funding mechanisms is associated with greater costs, including increased administrative monitoring and higher reporting costs. This tension between the stability benefits and the administrative burdens of diversification became a defining theme in the literature, with researchers increasingly recognizing that diversification is not an unqualified good. For example, Frumkin and Keating (2011) found that organizations with highly concentrated and specialized forms of revenue experienced significant benefits in the form of lower administrative and fundraising expenses, although these savings were associated with greater exposure to changes in the organization's financial position.

Given the tension shown between risk mitigation and additional administrative burden, several researchers have sought to synthesize the literature through meta-analytic methods. The meta-analysis by Hung and Hager (2019) of 40 original studies that reported 296 statistical effects found a small, positive but statistically significant association between revenue diversification and nonprofit financial health, while also showing that the granularity of diversification measurement influences the size of the effect. A subsequent bibliometric and meta-analysis by Lu et al. (2019) reached complementary conclusions but with an important nuance: that diversification had little effect on financial vulnerability and a slightly negative effect on financial capacity, attributing the inconsistency in results in existing studies to differing theoretical assumptions and calling for more discussion of the assumptions and effectiveness of revenue diversification among nonprofit scholars. Together, these syntheses confirm that, while the overall direction of the relationship between diversification and financial health is modestly positive, the effect is neither large nor universal, and methodological and contextual factors play a substantial role in shaping outcomes.

More recent scholarship has examined the limits of diversification as a financial resilience strategy, particularly during periods of systemic economic stress. Choi (2025) found that while revenue diversification effectively reduced revenue volatility under normal conditions, this effect did not persist during and after the 2008 Great Recession, although certain revenue sources were shown to be more efficient in reducing volatility during economic downturns. Furthermore, Searing and St. Clair (2025) examined the relationship between revenues and business cycle risk, finding that a nonprofit's exposure to the business cycle

depended on the component revenue types. These efforts raise important questions about the conditions under which diversification strategies are most effective and whether the types, in addition to the number, of revenue streams matter during crises.

### 3 Revenue Diversification and Portfolio Theory

As noted above, much of the public finance and financial management literature relies on measures of diversification. We can consider a traditional measure of revenue diversification, the Herfindahl–Hirschman Index (HHI), to understand the nature of information provided by these measures. Typically calculated using formula (1) or (2) below, these depend on the share each revenue stream contributes to total revenue, using the sum of these squared proportions to identify the diversification of this structure and ranging from  $1/n$  to 1 or 0 to 1, with the upper bound representing maximum concentration (1) or diversification (2).

$$HHI = \sum_{i=1}^n R_i^2 \quad (1)$$

$$RDI = \frac{(1 - \sum_{i=1}^n R_i^2)}{(n - 1)/n} \quad (2)$$

This measure depends on the fundamental logic of diversification; an organization that relies on a single source of revenue has a risk equal to the size of any disruption to that stream, while an organization reliant on multiple unassociated revenue sources will reduce its risk directly in proportion to the number of sources (3).

$$\frac{1}{n}(\sigma_{r1}^2) \quad (3)$$

These measures provide little information on the composition of the portfolio unless  $HHI = 1/n$  or  $RDI = 1$ , in which case the measure indicates that each source contributes equally to the revenue structure. Therefore, they incorporate almost no information relevant to the logic of revenue diversification, such as the revenue streams included or their relative contributions, the risk of these streams, or how they change together. Although the literature on revenue diversification suggests that concentration or diversification may be beneficial, work relying on HHI-like measures contributes little to our understanding of the impor-

tance of individual revenue sources or how they might be utilized together to best contribute to a revenue structure.

To better understand the importance of revenue sources and structures in public sector revenue management, methods that incorporate more information are required. Methods such as MPT or its successors, by analyzing stream's individual and interdependent characteristics, can provide useful information about the nature of individual revenue sources and their contribution to the performance of the organization's revenue structure.

Markowitz's MPT (1952) focused on the structure of investment portfolios and was based on risk aversion, positing that investors would prefer the least risky portfolio available for any given level of return, despite their varied risk tolerances. In this way, Markowitz understood investment choice as an optimization problem, minimizing risk at each level of return to identify an efficient frontier comprising optimal portfolios. Although later developments, sometimes known as post-modern portfolio theory (PMPT), have modified these methods to improve risk measurement, they still rely on the same fundamental logic.

Although much of the literature on municipal revenue focuses on diversification or concentration, we draw on portfolio theory to apply portfolio models directly to municipal revenue structures. We base this on the expectation that municipal governments will also have specific preferences regarding revenue growth and a desire to minimize risk. We begin with traditional models, but also apply post-modern techniques to examine how these adapted approaches influence results. This results in a comparison between Markowitz's symmetric MPT models and the asymmetrical measures of PMPT (Chen and Yuan, 2016). Ultimately, the results demonstrate the importance of considering revenue characteristics when analyzing revenue structures and how the measures used to represent those characteristics influence the identification of theoretically ideal portfolios, particularly risk measures.

## 4 Methodology

Modern and post-modern approaches to portfolio analysis rely on optimization, identifying combinations of elements, whether assets or revenue streams, that maximize one criterion relative to another. In most cases, this depends on minimizing a risk measure with respect to a desired characteristic (typically, asset return or growth). Given the definitions of these criteria, efficient portfolios are generated by assessing the attributes of numerous individual elements and the prevalence of each component within the whole (as described below). In the case of municipal revenue structures, this requires analyzing each revenue source over time and relative to other sources.

In this case, the desired characteristic is defined as revenue growth. This is measured as in (4), which is

equivalent to the first difference in the natural logs of annual revenue for each stream. These are used to calculate the average growth for each stream (5).

$$r_{i,t} = \ln\left(\frac{P_{i,t}}{P_{i,t-1}}\right) \quad (4)$$

$$\bar{r}_i = \frac{1}{N} \sum_{T=1}^M r_{i,t}, \quad i = \{1, \dots, T\} \quad (5)$$

We rely on this measure of revenue stream growth to calculate the total growth of any possible revenue structure, which is a function of each stream's average growth (5) and the proportion each stream contributes to the overall revenue structure (6). This represents the historic average growth of any possible combination of revenues (7).

$$x = \begin{bmatrix} rp_1 \\ . \\ rp_n \end{bmatrix} \quad (6)$$

$$\mathbb{E}[r_x] = x^T \cdot R = \sum_{i=1}^N x_i \mathbb{E}[r_i] \quad (7)$$

Although growth has been consistently measured in portfolio theory, risk measurement has varied over time and according to technique. Markowitz's original measure was symmetric, fundamentally relying on the variance of each element. This is represented by the variance of the revenue sources in our analysis (8), which treats deviations above and below the target (the mean by definition, although this can be adjusted) as equivalent.

$$\sigma_i^2 = \sigma_{ii} = \frac{\sum (r_i - \bar{r})^2}{N} \quad (8)$$

The inclusion of upside variance in the measure of risk has been a long-standing point of contention in the portfolio literature (Markowitz, 1999). Given this, other scholars have partitioned this variance into

semivariance, most often focusing on the downside risk, as in (9), where only deviations below the mean contribute to the measure of risk. Other targets can also be substituted by changing the constraint on the deviations that contribute to the semivariance, for example, by including only negative growth (10).

$$\frac{\sum(r_i - \bar{r})^2}{N} \quad \text{if } r_i - \bar{r} < 0 \quad (9)$$

$$\frac{\sum(r_i - \bar{r})^2}{N} \quad \text{if } r_i < 0 \quad (10)$$

This paper takes a slightly different approach: rather than partitioning the variance and excluding some of it, we propose measuring variance as revenue growth relative to a target and then weighting the upside and downside risk by risk tolerance. To do so, we establish a growth target ( $g$ ) and divide the distribution of the first differences into upside deviations ( $\delta_U$ ), above the target (11), and downside deviations ( $\delta_D$ ), below it (12).

$$\delta_U = \sum(r_i - g)^+ \quad (11)$$

$$\delta_D = - \sum(g - r_i)^+ \quad (12)$$

Including the parameter for a revenue growth target is helpful for assessing municipal revenue portfolios, as municipalities may want to understand their revenues relative to their past performance, the need to ensure stability, macroeconomic indicators, or averages of some comparison group. This preference may lead them to use a mean, zero value, economic indicator, or cross-sectional average as a target, respectively.

These elements are then weighted according to the tolerance for each type of risk, with a target set at the average return and a complete tolerance for upside risk equivalent to (9), and the same risk tolerance and a zero growth target equivalent to (10).

In addition to the revenue growth target parameter, our method adds weights (here  $\omega_U$  and  $\omega_D$ ) to the risk calculation (13), so that the upside and downside risk can be set to contribute unequally to the total risk measure.

$$W\sigma_{ii} = \frac{\omega_U \cdot \delta_U^2 + \omega_D \cdot \delta_D^2}{N} \quad (13)$$

In this way, the traditional asymmetric risk measure, semivariance (9), can be understood to be a special case of our method, when upside risk is not a concern (14). Markowitz's symmetrical measure, the variance, can also be considered a special case, as it (8) gives equal weight to the upside (11) and downside risk (12), and uses mean growth as the target (15).

$$\omega_U = 0 \quad (14)$$

$$g = \bar{r} \quad (15)$$

To assess the performance of revenue structures, associations among revenue streams must also be considered. Traditionally, these have been measured as covariances (16). However, weighted first differences can also be used as components of measures of association between revenue streams over time; they do so in the form of weighted covariances. These are comprised of the average of the summed products of the weighted differences from the target growth (17). This allows our measure to scale with the weighted variance, in accordance with tolerances for upside and downside risk.

$$\sigma_{i,j} = Cov(i,j) = \frac{1}{M} \sum_{t=1}^M (r_{i,t} - \bar{r}_i) \cdot (r_{j,t} - \bar{r}_j) \quad (16)$$

$$W\sigma_{i,j} = W \cdot Cov(i,j) = \frac{1}{M} \sum_{t=1}^M (\omega_U \cdot \delta_{U(i,t)} + \omega_D \cdot \delta_{D(i,t)}) \cdot (\omega_U \cdot \delta_{U(j,t)} + \omega_D \cdot \delta_{D(j,t)}) \quad (17)$$

The product of the portfolio weights (6) and the weighted covariance matrix (18) is used to calculate the total risk of the portfolio as reflected in (19).

$$W_{Sx} = \begin{bmatrix} W\sigma_{11} & \cdot & W\sigma_{n1} \\ \cdot & \cdot & \cdot \\ W\sigma_{1n} & \cdot & W\sigma_{nn} \end{bmatrix} \quad (18)$$

$$W\sigma_p = \sqrt{x^T W_{Sx} x} = \sqrt{\sum_{i=1}^N \sum_{j=1}^N X_i X_j \sigma_{ij}} \quad (19)$$

Return and risk measures are used in an optimization function that maximizes  $\Theta$ , the ratio of return to risk (20).

$$\max \Theta = \frac{\mathbb{E}[r_x] - c}{W\sigma_p} \quad (20)$$

The benefit of this formulation is that the risk-weighting parameters and target can be set at any desired level, allowing efficiency to be defined flexibly. This enables the use of varied growth targets and risk tolerances to understand the performance of revenue structures. This can be applied across multiple conditions or to particular types of municipalities to compare how these targets and tolerances might influence optimal portfolio composition; we focus on the importance of risk tolerances below to demonstrate how varied preferences on a single dimension could influence the preferred revenue structure.

## 5 Data

We rely on revenue data from Minnesota municipalities for this analysis, including all municipalities with reported revenue from 2003 to 2022. This data is provided by the Office of the State Auditor and the Government Information Division. This provides data on approximately 857 different municipalities in total. Select municipalities are removed from our analysis for a variety of reasons, including inability to merge with relevant population/city type data, and in certain instances, smaller municipalities being absorbed by neighboring larger cities, leading to the smaller cities dropping out of the dataset. However, these problems occur in a negligible number of municipalities, all of which have small populations. Our final dataset contains 828 municipalities.

We incorporate additional municipal demographic data to supplement our revenue figures. Our popu-

lation data are sourced from the Minnesota State Demographic Center, Metropolitan Council, and the U.S. Census Bureau; these were collected and downloaded via the Minnesota State Demographic Center website. To price-adjust the revenue data, we use the US Bureau of Labor Statistics' Consumer Price Index (CPI) to adjust to 2025 USD.

## 6 Results

We examine Minnesota municipalities in several ways, identifying: (a) optimal portfolios for individual municipalities, (b) the average portfolios for groups of municipalities (classified by population and density), and (c) the average optimal portfolio based on data from all municipalities. In this way, we examine the variations in optimal portfolios.

For (a), we have selected nine individual municipalities for analysis based on population and city type (6.3 below). We use three discrete population groups: large ( $> 50,000$ ), medium (10,000 to 50,000), and small ( $< 10,000$ ). 'City type' is defined as the population density using estimates of zip code-level data from [Kolko \(2015\)](#). There are three different designations: urban, suburban, and rural. If a city has multiple zip codes with different designations, the city is assigned the designation with the highest density (urban  $\succ$  suburban  $\succ$  rural). Examining these cases allows us to compare the importance of risk tolerance when the revenue and service profile are constant, as well as assessing how the service profile of cases might influence results.

We also use size and city type categories to examine the optimal portfolios for grouped municipalities for (b) (section 6.4 below). Table 1 presents the characteristics of each population/city-type group, as well as Minneapolis and St. Paul, the number of cities in each category, and the total observations in our dataset for each category. Examining these categories provides more information about specific cases, as we can understand how they might differ from their peers, while also providing more information about contextual factors such as size, location, and service profile.

Finally, we examine the average optimal portfolio for all municipalities in Minnesota in (c) (6.5 below), demonstrating variations between cases, categories, and the overall average, highlighting the need to consider the characteristics of the government entities included when generating average 'optimal' structures.

We perform two optimizations for each city and category. The first relies on symmetric measures of variance and covariance (with variance calculated as in Equation 8), while the second uses an asymmetric measure that includes only downside risk (with upside risk set to zero, as in Equation 14). This allows us to assess how portfolio preferences might change, examining the implications of utilizing either a traditional measure or one that focused exclusively on downside risk.

## 6.1 Descriptive Analysis of Municipalities and Groups

We begin our analysis by examining the revenue structures of Minnesota municipalities, ordered by our aforementioned type and size categories. This includes Minneapolis, the state’s major urban center, as well as St. Paul, the state capital, the other half of the ‘Twin Cities’ moniker, and the second-largest city in the state; Minneapolis generates much more revenue from property and sales taxes, whereas St. Paul is more reliant on state aid. These cities are analyzed alone as they differ greatly in size from other ‘large’ municipalities in the state.

It also includes Duluth, a major economic hub at the western terminus of Lake Superior and the Great Lakes (our lone ‘Large Urban’-designated city), which is isolated from other population centers. There are no communities classified as ‘Mid Urban’ or ‘Small Urban.’ Rochester is a large suburban municipality in southeastern Minnesota and a center for healthcare. Edina is an affluent suburb of Minneapolis with a population of almost 50,000, categorized as ‘Mid-Suburban’; while Lake Elmo is a small suburb of St. Paul. St. Cloud is the lone ‘Large Rural’ city of around 70,000 with a large state university; while Bemidji is a city of about 15,000 in the north that acts as a regional hub and is in the ‘Mid Rural’ group; and Wadena, a local hub of about 4,500 in rural west-central Minnesota, represents the cities in the ‘Small Rural’ group. Table 1 (below) lists our groups and municipalities, along with brief descriptions of these categories.

**Table 1:** *Population/City Type Group Counts*

Group	Municipality	Population	City Type	# of Cities	# of Observations
-	Minneapolis	436,934 (2022)	Urban	1	20
-	St. Paul	310,992 (2022)	Urban	1	20
-	Duluth	89,000 (2022)	Urban	1	20
Large Suburban	Rochester	> 50,000	Suburban	16	263
Mid Suburban	Edina	[10,000,50,000)	Suburban	56	1,030
Small Suburban	Lake Elmo	< 10,000	Suburban	60	1,139
Large Rural	St. Cloud	> 50,000	Rural	1	20
Mid Rural	Bemidji	[10,000,50,000)	Rural	29	529
Small Rural	Wadena	< 10,000	Rural	669	13,314

**Alt text:** Table reporting the population/city type group breakdown, and their representative cities that will be further analyzed in the paper. Minneapolis, St. Paul, Duluth, and St. Cloud (representing the ‘Large Rural’ designation) are standalone categories and have their specific populations listed, and all have 20 observations in total, one for each year. The large suburban designation has 16 cities that meet its criteria, with 263 observations total in the final data. The mid-suburban has 56 cities, good for 1,030 total observations. The small suburban category contains 60 cities and 1,139 observations. The mid-rural group has 29 cities and 529 observations, with the small rural group being the largest, with 669 cities and 13,314 observations

total in the final data.

## 6.2 Assessing Group Classification using Within-Group Variance

A first step in assessing the appropriateness of the previously highlighted groups is to examine the variation that occurs within the groupings.

**Table 2:** Summary Statistics of Expected Returns from Optimal Portfolio

Group	# of Cities	Min	25 <sup>th</sup> Pctile	Mean	Median	75 <sup>th</sup> Pctile	Max
Minneapolis	-	-	-	0.034	-	-	-
St. Paul	-	-	-	0.058	-	-	-
Duluth	-	-	-	0.379	-	-	-
Large Suburban	263	-0.214	-0.059	-0.031	-0.003	0.029	0.353
Mid Suburban	1,030	-0.380	-0.064	-0.003	0.009	0.056	0.551
Small Suburban	1,139	-0.617	-0.168	-0.042	0.007	0.197	0.792
Large Rural (St. Cloud)	1	-	-	0.008	-	-	-
Mid Rural	529	-0.479	-0.057	0.027	0.006	0.098	0.325
Small Rural	13,314	-0.407	-0.033	-0.001	0.003	0.029	0.655

**Alt text:** Table reporting summary statistics of expected returns from the optimal portfolio by city group, including number of cities in the group, minimum, 25th percentile, mean, median, 75th percentile, and maximum. For individual cities (Minneapolis, St. Paul, Duluth, and St. Cloud), only mean returns are reported: 0.034, 0.058, 0.379, and 0.008, respectively. Among the suburban groups, large suburban areas (n=263) have a mean return of -0.031 (median -0.003; range -0.214 to 0.353), mid suburban areas (n=1,030) have a mean of -0.003 (median 0.009; range -0.380 to 0.551), and small suburban areas (n=1,139) have a mean of -0.042 (median 0.007; range -0.617 to 0.792). Among the rural groups, mid rural areas (n=529) have a mean of 0.027 (median 0.006; range -0.479 to 0.325), while small rural areas (n=13,314) have a mean of -0.001 (median 0.003; range -0.407 to 0.655).

**Table 3:** Most Common Largest Revenue Category in Optimal Portfolio

Group	# of Cities	Category	Proportion
Minneapolis	-	Charges from Traditional Services	-
St. Paul	-	State/Local Governmental Aid	-
Duluth	-	Charges from Enterprise Services	-
Large Suburban	263	Property Taxes	0.69
Mid Suburban	1,030	Local Sales Tax	0.53
Small Suburban	1,139	Federal Inter-Governmental Revenue	0.27
Large Rural (St. Cloud)	1	Property Taxes	-
Mid Rural	529	Property Taxes	0.38
Small Rural	13,314	State/Local Governmental Aid	0.22

**Alt text:** Table showing the most common largest revenue category by city group, along with the number of cities and proportion. Only the dominant category is reported for Minneapolis, St. Paul, Duluth, and St. Cloud. In the suburban group, most prevalent category for large suburban areas is property taxes (69% of cities), mid suburban areas rely on local sales tax (53%), and small suburban areas on federal intergovernmental revenue (27%). Among rural jurisdictions, property taxes are the most common (38%) for mid rural, and small rural has state/local governmental aid (22%).

There may be an association between the size and density of the city and the propensity of municipalities in a group to share a common revenue source that serves as the primary contributor to an optimal revenue structure (as seen in Table 3). Lower population and density correspond to a lower proportion of municipalities sharing a dominant revenue category, which may be a function of the diversity of funding models available to smaller municipalities or could be attributed to other factors, such as the number of cases in each group.

Although optimal portfolios are more similar in larger and denser categories, the structure of optimal portfolios seems to increase in variation with size and density, with the mean and median contribution of the largest revenue category being smaller in the largest and densest categories and smallest in the smallest and least dense (displayed in Table 4).

**Table 4:** *Summary Statistics of Maximum Revenue Category Weight from Optimal Portfolio*

Group	# of Cities	Min	25 <sup>th</sup> Pctile	Mean	Median	75 <sup>th</sup> Pctile	Max
Minneapolis	-	-	-	0.377	-	-	-
St. Paul	-	-	-	0.379	-	-	-
Duluth	-	-	-	0.309	-	-	-
Large Suburban	263	0.195	0.263	0.386	0.322	0.466	0.777
Mid Suburban	1,030	0.206	0.301	0.424	0.408	0.516	0.690
Small Suburban	1,139	0.223	0.340	0.476	0.453	0.590	0.813
Large Rural (St. Cloud)	1	-	-	0.390	-	-	-
Mid Rural	529	0.209	0.253	0.334	0.325	0.400	0.526
Small Rural	13,314	0.150	0.325	0.439	0.412	0.529	0.984

**Alt text:** Table showing summary statistics of maximum revenue category weight across city groups. Minneapolis, St. Paul, and Duluth report only mean values (around 0.31–0.38). Suburban groups (large, mid, small) show increasing means from 0.386 to 0.476 with corresponding distributions. Rural groups include St. Cloud, mid rural (mean of 0.334), and small rural (mean of 0.439) with wider ranges up to 0.984.

### 6.3 Selected Individual Municipal Portfolio Analysis

Examining our example cities and groups, we observe unique optimal structures and differences arising from the risk measure used in the optimization. We first analyze our selected cases and then the groups of municipalities. Each figure below represents 100 optimizations as in (20), using either symmetric or asymmetric calculations.

**6.3.1 Minneapolis**—Examining the symmetrical and asymmetrical portfolios for Minneapolis reveals the importance of the definition of risk in identifying efficient revenue structures. Although a symmetrical definition of risk indicates a low risk and return option, with negative growth (inflation-adjusted) and low risk, as well as a maximum return of over .06 (as seen in Figure 1, Panel a), the optimization using only downside risk identifies structures with the lowest possible return above .03 and maximum returns of over .06 (as seen in Figure 1, Panel b).

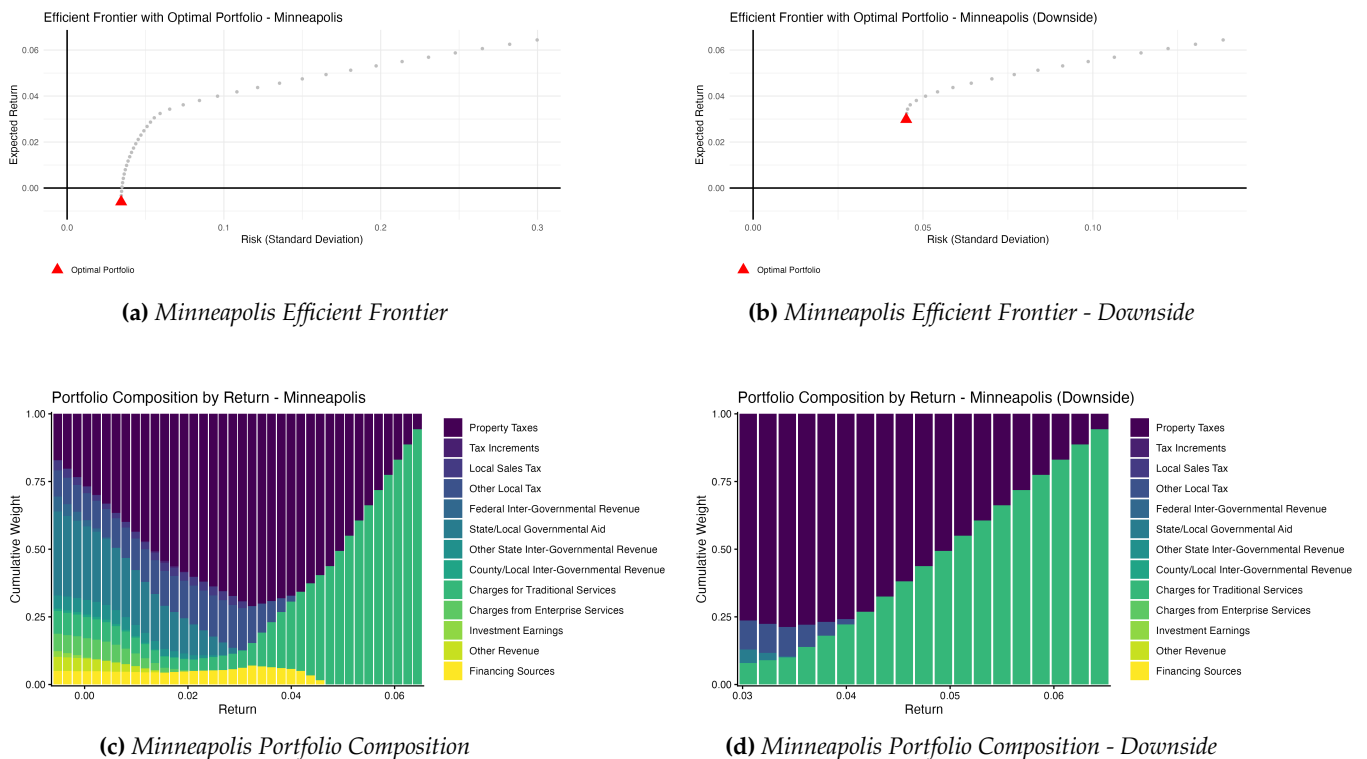
This result suggests that including upside risk in revenue analysis may misrepresent the desirability of certain revenue sources and that prioritizing them in strategic revenue management may be counter-productive. In the case of Minneapolis, it results in portfolios with charges for enterprise services, federal inter-governmental revenue (IGR), financing, and non-aid state IGR; as well as greater proportions of other local taxes and state local government aid (Figure 1, Panel c), than would be considered optimal using an asymmetric definition of risk, where property taxes and charges for services look more attractive (Figure 1, Panel d).

In the case of Minneapolis, the composition of the symmetric portfolio, ranging from the minimum to 0.03 growth, is likely to be particularly problematic if it does not reflect the actual risk tolerance. In these cases, it includes streams that compromise growth without reducing the potential for downside risk and would thereby limit revenue growth and undermine revenue management. For returns above 0.03, both financing and other local taxes are more attractive with a symmetrical measure than under an asymmetrical one, as they are either absent or constitute smaller components of the efficient asymmetrical portfolio, respectively.

In the asymmetric portfolio, we observe a less diverse revenue structure, with a substitution of property tax for service charges as risk tolerance increases. This is an intuitive finding for Minneapolis, which has experienced fairly consistent growth in property tax revenue. This has increased each year of the study, except in 2009, 2011, and 2014. Minneapolis has relied on property taxes for about one-quarter of its revenues from 2003 to 2022. Both portfolios also identify the potential for other local taxes to reduce risk in low-risk-tolerance optimal portfolios; Minneapolis relies on franchise fees, hotel and motel taxes, special assessments, and licenses and permits as the primary sources of revenue in this category.

Of course, we do not mean to suggest that positive deviations should be ignored when calculating risk; we recognize that budget ratcheting or other negative consequences might result from unanticipated and unnecessarily inconsistent revenue growth. We would expect governments to consider weighing the potential harm from these deviations differently relative to downside risk, and that these more nuanced decisions might inform strategic revenue management. We also do not expect the theoretically optimal portfolios to be viable in practice for Minneapolis; rather, we only expect them to indicate which streams are likely to be reliable and complementary when considering trade-offs between risk and return.

**Figure 1: Minneapolis Comparing Symmetrical and Asymmetrical Efficient Portfolio Frontiers and Compositions**

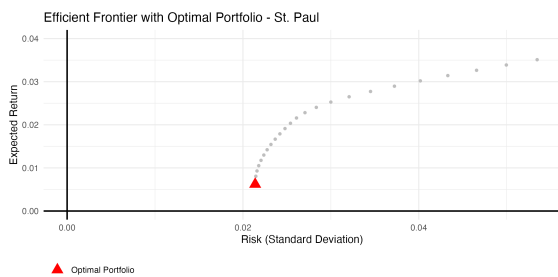


**Alt Text:** The top row of Figure 1 shows the efficient frontier plots for the symmetric and downside portfolios. The expected return of the symmetric portfolio is -0.6%, but is 2.98% in the downside case. Across returns, the portfolio composition becomes increasingly reliant on property taxes before peaking at around 3% before charges for traditional services become the majority category, increasing in lockstep with the return rate. In the downside portfolio, a similar trend emerges as an increase in return results in the movement from property taxes to charges for traditional services as an almost 1:1 rate.

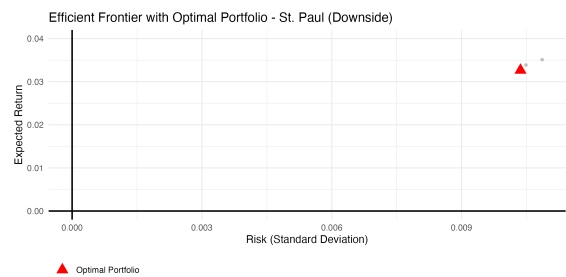
**6.3.2 St. Paul**—The City of St. Paul further illustrates the importance of risk definitions. Comparing the symmetrical and asymmetrical portfolios for the city, we again see quite different efficient frontiers (Figure 2, panels **a** and **b**), as well as drastically different revenue structures along this frontier (Figure 2, panels **c** and **d**). Although symmetrical measures include charges for services, federal IGR, and Tax Increment Financing (TIF) funding, and returns from 0.00625 to 0.035, the asymmetrical portfolio identifies only two options: one comprised of a small amount of local sales tax and property taxes, with a return of 3.35%, and the other entirely of property taxes, with a return of 3.5%. These two sources have less downside risk and offer a return comparable to the maximum identified using a symmetric measure of risk.

We do not intend to suggest that either of these portfolios is a practical revenue structure for St. Paul; we only propose that using an asymmetric measure would alter the revenues identified as most desirable in this case, as well as the potential benefit of diversifying the city’s revenue structure. Multiple revenue streams are less appealing when risk is defined asymmetrically, whereas a more concentrated revenue structure may be considered more acceptable, depending on its composition. We note that property tax is the largest element of St. Paul’s actual revenue structure during this period, while local sales taxes are included in a set of sources contributing an average of 4% to 10% to St. Paul’s revenues from 2003 to 2022 (along with TIFs, other taxes, county local government aid, and other state IGR, and charges for services).

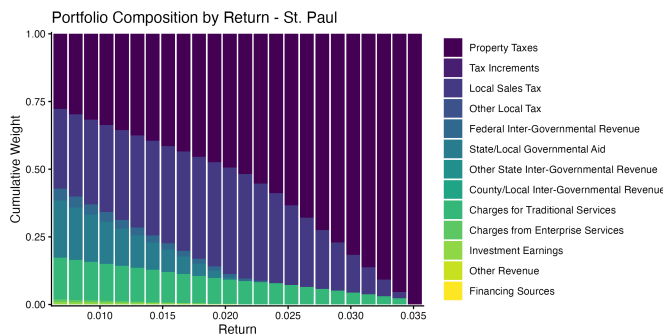
**Figure 2: St. Paul Comparing Symmetrical and Asymmetrical Efficient Portfolio Frontiers and Compositions**



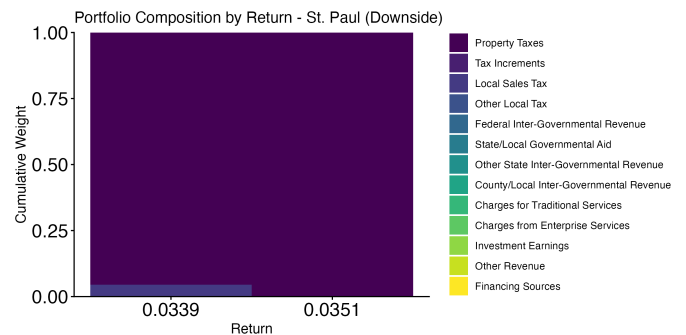
**(a) St. Paul Efficient Frontier**



**(b) St. Paul Efficient Frontier - Downside**



**(c) St. Paul Portfolio Composition**



**(d) St. Paul Portfolio Composition - Downside**

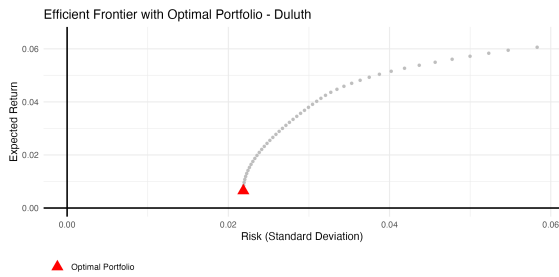
**Alt text:** The top row of Figure 2 shows the efficient frontier plots for the symmetric and downside portfolios. The expected return of the symmetric portfolio is 0.6%, but is 3.27% in the downside case. Across returns on the bottom row, the symmetric portfolio sees property taxes gradually increase in share until it is the entirety of the portfolio. The downside portfolio only has two categories at any given rate of return: Local sales tax and property taxes.

**6.3.3 Duluth**—Despite substantial differences in the size and economies of Duluth and the Twin Cities, the changes in optimal structures identified when using symmetric or asymmetric risk measures show some similarities. Several revenue streams appear in symmetrically efficient portfolios, but are omitted from the frontier portfolios when optimization relies on asymmetric risk. This includes state IGR, other revenues, and charges for traditional services. The degree to which a theoretically optimal portfolio for Duluth relies on streams like financing services, local sales tax, and state and local government aid also varies substantially depending on the measure of risk, with the first two dominating the asymmetric frontier portfolios.

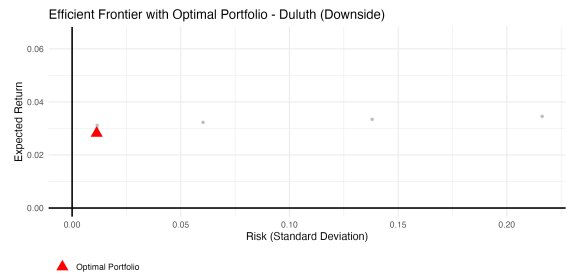
We note that Duluth has relied on local sales taxes almost as much as on property taxes during this period (10.5% compared to 12%, respectively). State local government aid has been a substantial source of revenue for the city, but declined as a share of revenue from 2019 to 2022 and experienced relatively little growth, with multiple periods of near stagnation (2010 to 2013; 2014-2017).

Asymmetric portfolios offer returns of about 0.03 with risk below 0.015, reducing the risk required to achieve the same growth as symmetric portfolios, which have a corresponding risk of about 0.025. This suggests that the inconsistent growth of these sources leads them to be omitted from symmetrically efficient portfolios. Inadvertently weighing upside and downside risk equally, when downside risk is more important, could lead organizations to pursue flawed revenue strategies. **Alt text:** The top row of Figure 3 shows the efficient frontier plots for the symmetric and downside portfolios. The expected return of the symmetric portfolio is 0.6%, but is 2.82% in the downside case. Across returns in the symmetric portfolio, an increase in return causes property taxes to gradually increase in portfolio share as state/local government aid and local sales tax decrease in turn. The downside portfolios have the opposite effect; an increase in return causes the share of property tax to be lessened by financing sources.

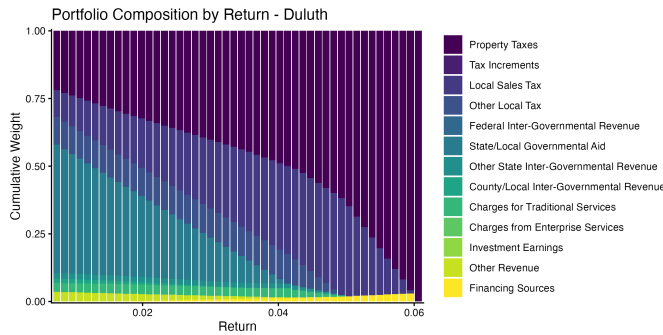
**Figure 3: Duluth Comparing Symmetrical and Asymmetrical Efficient Portfolio Frontiers and Compositions**



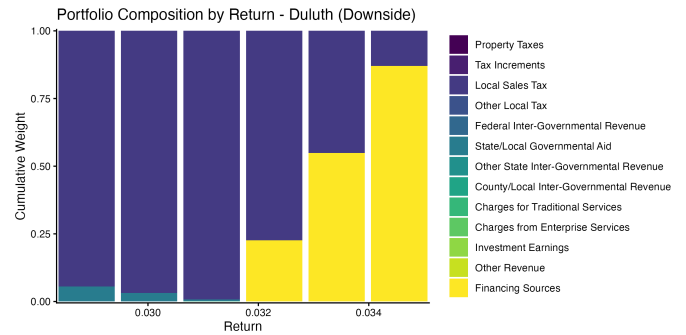
**(a) Duluth Efficient Frontier**



**(b) Duluth Efficient Frontier - Downside**



**(c) Duluth Portfolio Composition**



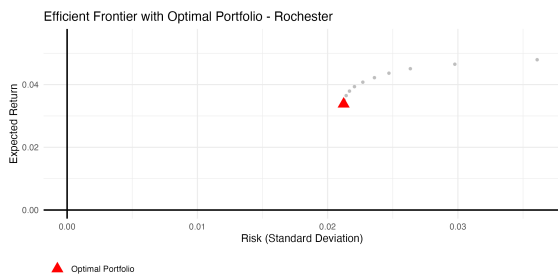
**(d) Duluth Portfolio Composition - Downside**

**6.3.4 Rochester**—The optimal frontier revenue structures for Rochester are largely made up of property tax revenue, regardless of whether risk is measured symmetrically or asymmetrically. This would not be surprising, given the tremendous growth in property tax revenue in the city, from \$22.5M in 2003 to \$86.7M in 2022, as well as the remarkable consistency of this growth (outside of 2013-2014); the city has relied on property taxes for 27% of its revenue during this span.

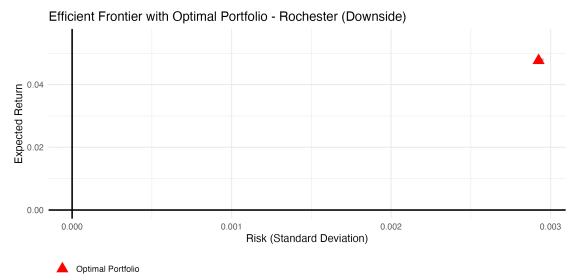
When the upside risk is eliminated from the calculations, property tax revenue is over 98% of the revenue in a 4.8% return portfolio (Appendix Table A4); Figure 4, Panels **b** and **d**). No optimal symmetric portfolio offers more return than this asymmetric portfolio (Figure 4, Panels **a** and **c**). This suggests that charges for services, financing sources, state and local government aid, and tax increments contribute more to upside risk than property taxes for Rochester. As in our previous cases, optimizations relying on symmetric measures would select a more diverse portfolio than those relying on downside-only risk, while sacrificing growth to minimize upside variance.

**Alt text:** The top row of Figure 4 shows the efficient frontier plots for the symmetric and downside portfolios. The expected return of the symmetric portfolio is 3.38% and 4.47% in the downside case. Across returns, property taxes are the predominant revenue category in both the symmetric and asymmetric portfolios.

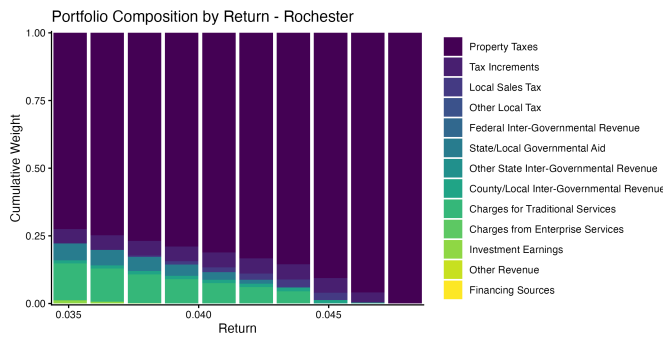
**Figure 4: Rochester Comparing Symmetrical and Asymmetrical Efficient Portfolio Frontiers and Compositions**



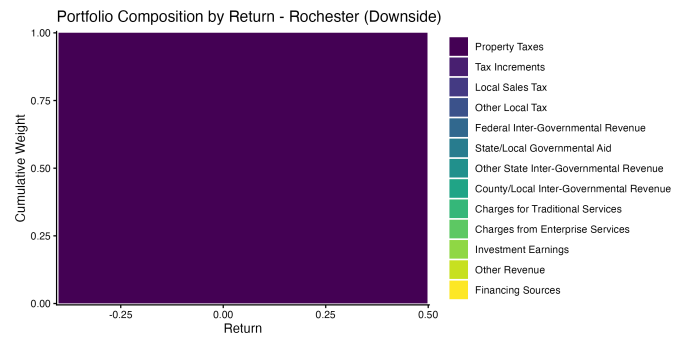
**(a) Rochester Efficient Frontier**



**(b) Rochester Efficient Frontier - Downside**



**(c) Rochester Portfolio Composition**

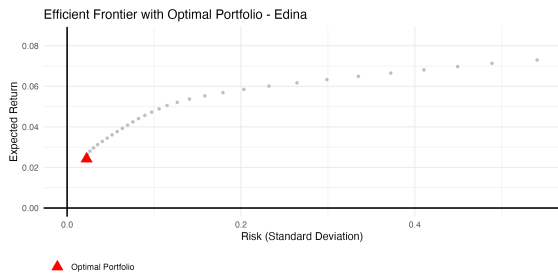


**(d) Rochester Portfolio Composition - Downside**

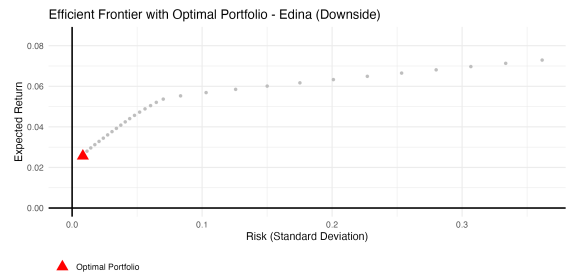
**6.3.5 Edina**—Edina, a mid-size suburban municipality near Minneapolis, has relatively similar symmetric and asymmetric portfolios, with only the federal IGR and financing sources categories substantially reduced in the semi-variance portfolios. Examining the optimal portfolios in Figure 5, panels c and d, the lowest risk and return portfolios are primarily comprised of charges for traditional services and property taxes, with property taxes being a slightly larger proportion of the asymmetric portfolios. These are large components of Edina’s actual revenue structure, typically around 45% (with roughly 40% of this property taxes during the study period). We observe that, for both symmetric and asymmetric portfolios, up to about 6% return, other local taxes generally become a larger proportion of revenue; these are primarily franchise and lodging taxes in Edina. These are gradually replaced by county/local intergovernmental revenue and other revenues in the highest return portfolios. Both of these are small components of Edina’s actual revenue structure and would seem to be unlikely to increase dramatically, reinforcing that theoretically optimal portfolios are suggestive rather than prescriptive.

**Alt text:** Figure 5’s top row illustrates the symmetric and asymmetric efficient frontiers, which are incredibly similar in this instance, having optimal returns of 2.42% and 2.56%, respectively. The bottom row displays the portfolio composition, where, like the efficient frontiers, they appear very similar in the allocation of optimal revenue categories. Property taxes, other local taxes, and charges for services are the

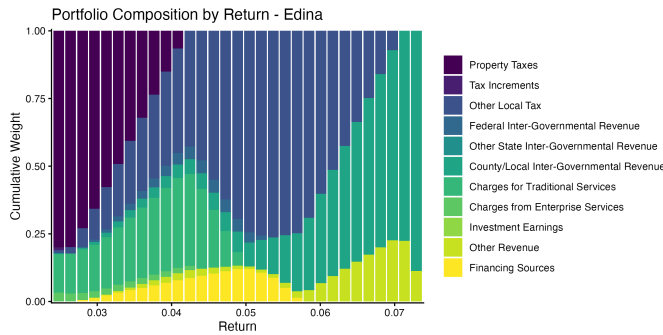
**Figure 5: Edina Comparing Symmetric and Asymmetric Efficient Portfolio Frontiers and Compositions**



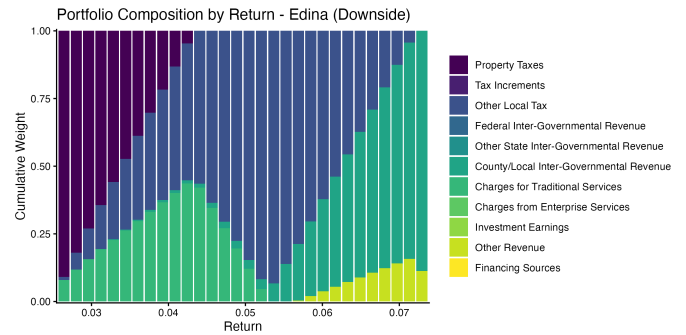
**(a) Edina Efficient Frontier**



**(b) Edina Efficient Frontier - Downside**



**(c) Edina Portfolio Composition**



**(d) Edina Portfolio Composition - Downside**

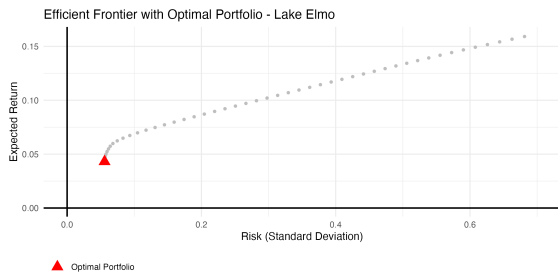
primary elements of these portfolios.

**6.3.6 Lake Elmo**—A small suburb on the east side of the Twin Cities, Lake Elmo relied on property taxes for approximately 40% of its revenue during the study period, which might be expected given the high degree of residential property in Lake Elmo (City of Lake Elmo Planning and Zoning Department, 2024). It also relied on financing (around 25%) and other local taxes (around 13%). Although the latter two are present to some degree in one or more of our symmetrical portfolios (Figure 6, panel c), neither is the primary component of any theoretically optimal symmetrical portfolio. Charges from enterprise services and investment earnings are also present in some symmetrically efficient portfolios.

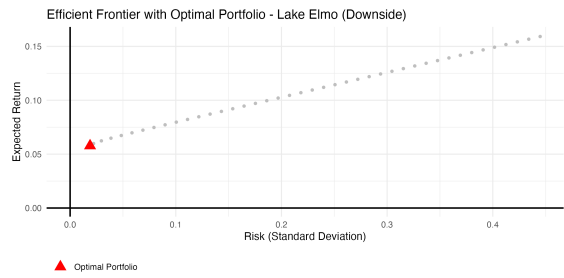
When downside variance contributes more to our definition of risk, most of these revenue types are eliminated from efficient structures (Figure 6, panel d); all portfolios are composed almost entirely of some proportion of property tax or other state intergovernmental revenue. Although it would seem unlikely that other state IGR would be a substantial source of revenue in Lake Elmo’s actual revenue structure, the asymmetrical portfolio demonstrates the balance of growth versus loss in the property tax revenue stream and highlights the relatively greater downside risk in Lake Elmo’s other local taxes, charges for enterprise services, and investment earnings. In general, we observe that the optimal asymmetric structure yields comparable revenue growth with less diversification, highlighting the potential for a symmetric risk

measure to overvalue some revenue streams.

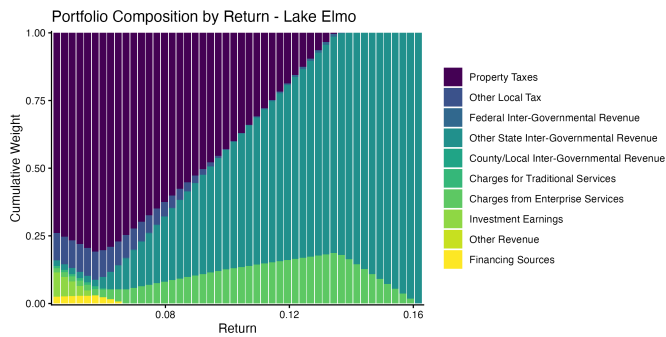
**Figure 6: Lake Elmo Comparing Symmetrical and Asymmetrical Efficient Portfolio Frontiers and Compositions**



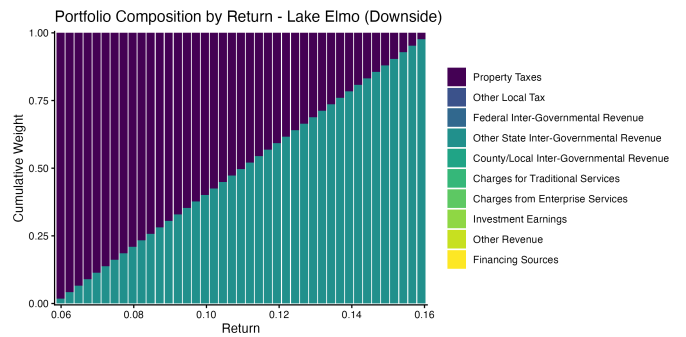
(a) Lake Elmo Efficient Frontier



(b) Lake Elmo Efficient Frontier - Downside



(c) Lake Elmo Portfolio Composition



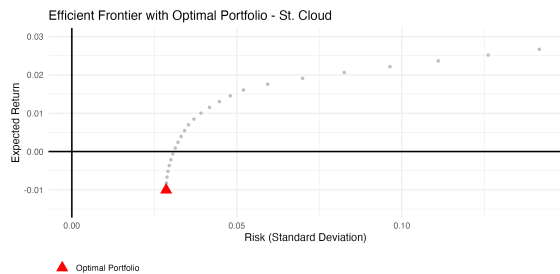
(d) Lake Elmo Portfolio Composition - Downside

**Alt Text:** Figure 6 shows the efficient frontier plots for Lake Elmo’s symmetric and downside portfolios, which have returns ranging from about 5% to just over 15%. Across the efficient frontier, both portfolio compositions initially rely heavily on property taxes and are replaced as return increases with other state IGR. In the downside portfolio, this replacement occurs at an almost 1:1 rate.

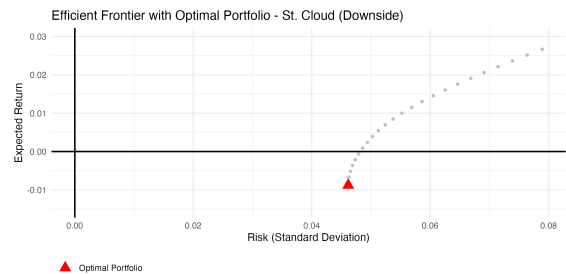
**6.3.7 St. Cloud**—Moving to rural municipalities, we examine three cases that stand out from prior examples, with substantial proportions of state and local government aid included in efficient revenue structures.

St. Cloud, for example, has both symmetrical and asymmetrical portfolios with greater proportions of state and local government aid (SLGA) than any prior case except Duluth. These are important revenues for St. Cloud, comprising an average of about 10.5% of its revenue during this period. Property tax is the other substantial element of both portfolios, although, relative to SLGA, it is a smaller component of efficient portfolios when only downside variance is included in our measure of risk. Other revenues are also present in both portfolios to similar degrees. Although charges for enterprise services, including fees for libraries, parks and recreation, and airports, as well as fines in St. Cloud, are present in the lowest-risk symmetrical portfolios, they drop out of asymmetrical frontier portfolios altogether, as do intergovernmental revenues and local sales tax (Figure 7, panels c and d). Comparing the efficient frontiers (Figure 7, panels a and b) demonstrates the tradeoff of accepting upside risk, as the slope of the portfolio return increases more rapidly relative to risk in the asymmetrical portfolio.

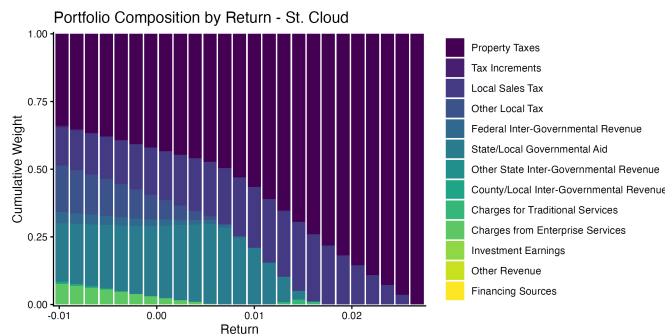
**Figure 7: St. Cloud Comparing Symmetrical and Asymmetrical Efficient Portfolio Frontiers and Compositions**



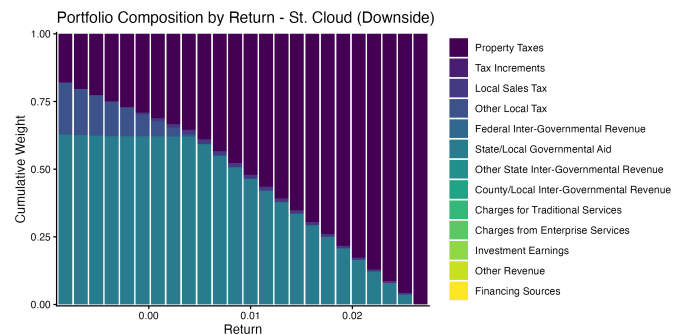
**(a) St. Cloud Efficient Frontier**



**(b) St. Cloud Efficient Frontier - Downside**



**(c) St. Cloud Portfolio Composition**



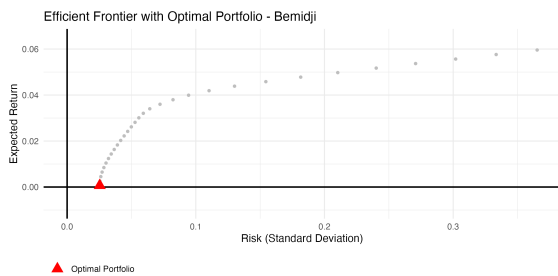
**(d) St. Cloud Portfolio Composition - Downside**

**Alt Text:** The top row of Figure 7 shows the efficient frontier plots for Lake Elmo’s symmetrical and downside portfolios, which have returns ranging from below 0% to over 2.5%. The symmetrical portfolio

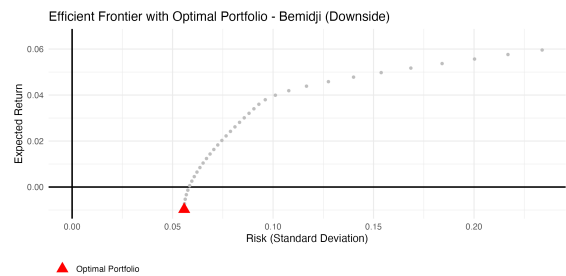
relies most heavily on property taxes, local sales tax, other local taxes, and state and local government aid, with the proportion of property tax increasing as return rises, especially as return approaches 1%. The asymmetric portfolio relies more heavily on other state IGR, with similar proportions of property taxes and other local taxes at the lowest level of return. Property taxes become more important as returns increase.

**6.3.8 Bemidji**—Bemidji’s optimal portfolios also include state and local government aid and property tax revenues, with a greater proportion of state and local government aid in the lower return asymmetrical portfolio, similar to St. Cloud. Bemidji’s actual revenue structure relies on both of these sources, with an average of 45% of revenue during this study. Other revenue is also present in both portfolios (Figure 8, panels c and d), with a smaller proportion of other revenue in the asymmetrical portfolio until it reaches maximum return. The composition of the asymmetric portfolios and the efficient frontier (Figure 8, panels a and b) suggest that an upside risk tolerance would facilitate higher returns and greater reliance on revenue from property taxes and other local revenue above 4% revenue growth, while the symmetric portfolio offers advantages below this threshold.

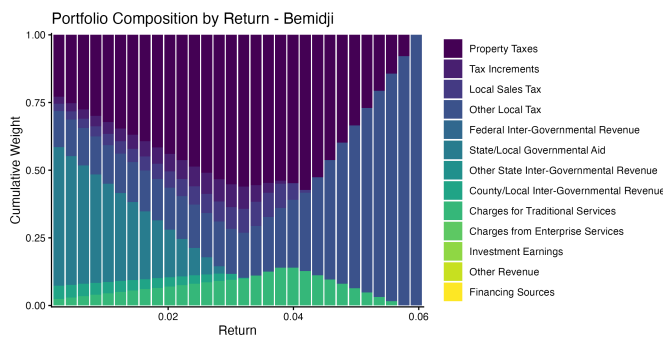
**Figure 8: Bemidji Comparing Symmetrical and Asymmetrical Efficient Portfolio Frontiers and Compositions**



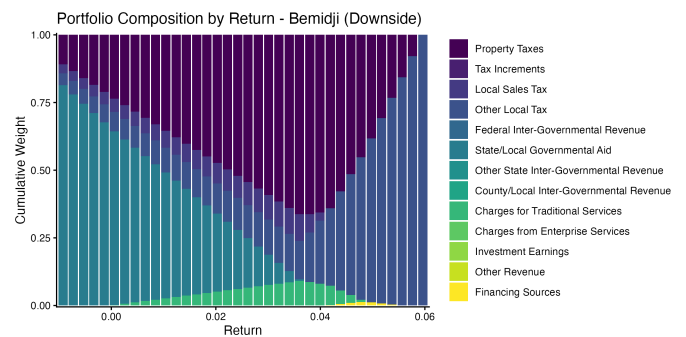
**(a) Bemidji Efficient Frontier**



**(b) Bemidji Efficient Frontier - Downside**



**(c) Bemidji Portfolio Composition**



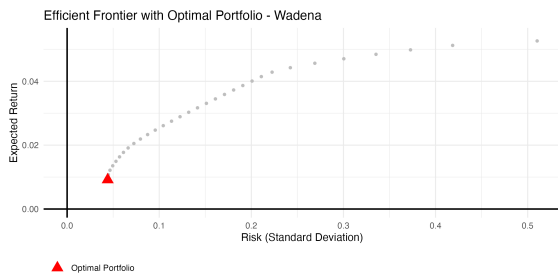
**(d) Bemidji Portfolio Composition - Downside**

**Alt Text:** The top row of Figure 8 shows the efficient frontier plots for Bemidji. Comparing the efficient

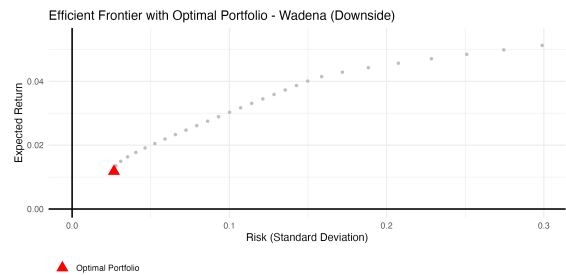
frontiers, the symmetric portfolio ranges from .07% return to 6%, with the lowest risk portfolio having a standard deviation of .0254. The lowest risk asymmetric portfolio has a return of nearly -1%, also reaching a maximum return of 6%. Portfolio compositions are similar, with property tax and state and local IGR comprising the largest share of the lowest risk portfolios, but with other local taxes increasing in proportion as return nears 4%. State and local IGR are the larger share of lowest return asymmetric portfolios.

**6.3.9 Wadena**—Wadena’s lowest risk symmetrical optimal portfolios include state and local government aid, property tax revenues, and charges for services, with rapidly decreasing proportions of property tax revenue and increasing proportions of county/local inter-governmental revenue as risk tolerance increases (Figure 9, panels c and d). Property taxes are absent from nearly all asymmetric portfolios, while state and local government aid decreases and county/local IGR increases with risk tolerance (Figure 9, panel d). Charges for enterprise services increase up to 4% and then decrease in both portfolio groups. Wadena’s actual revenue structure relies heavily on intergovernmental revenue and property taxes. Comparing the efficient frontiers (Figure 9, panels a and b) suggests that Wadena’s asymmetrically efficient portfolio yields higher returns for any given level of risk.

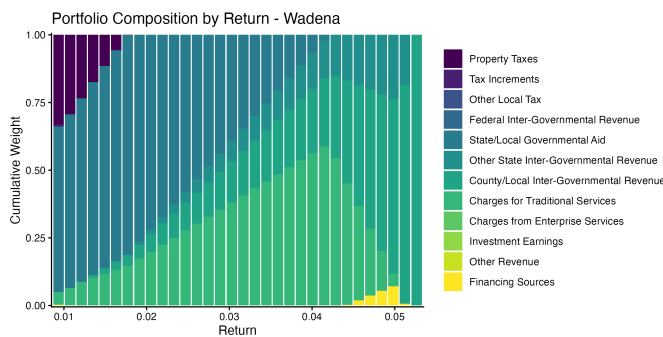
**Figure 9: Wadena Comparing Symmetrical and Asymmetrical Efficient Portfolio Frontiers and Compositions**



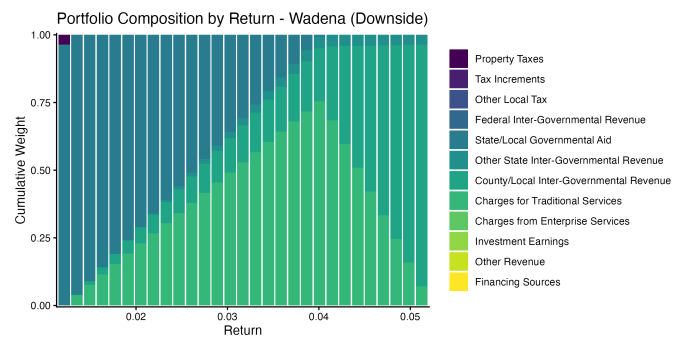
**(a) Wadena Efficient Frontier**



**(b) Wadena Efficient Frontier - Downside**



**(c) Wadena Portfolio Composition**



**(d) Wadena Portfolio Composition - Downside**

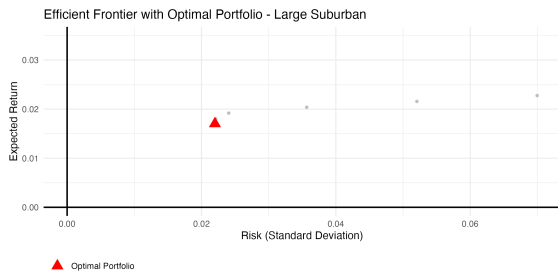
**Alt Text:** Wadena’s frontiers and portfolio compositions are displayed in Figure 9. The compositions are unusual, in that the majority of the lowest return portfolios are state and local government aid. This diminishes consistently until a return of about four percent, as the proportion of county and local IGR increases.

## 6.4 Municipal Category Portfolios

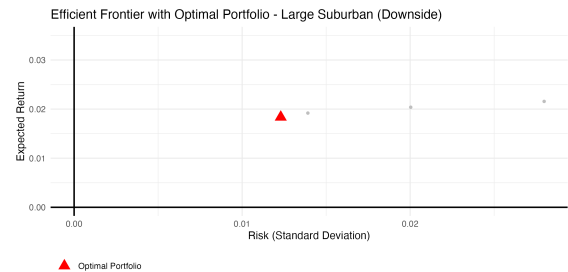
Examining the optimal portfolios of groups of municipalities also suggests that size and service profile are important. We see this when examining the differences between categories and the differences between these categories and the individual cases of the same type.

**6.4.1 Large Suburban**—The optimal portfolios for large suburban municipalities consist of charges for traditional services and property taxes with a small amount of other local taxes in the symmetric portfolio. The optimal asymmetric portfolio is identical to the symmetric portfolio at three levels of return. The optimal structure for this group of 263 municipalities differs from that in our large suburban case, Rochester, where property taxes dominate optimal portfolios that achieve higher growth than its peers.

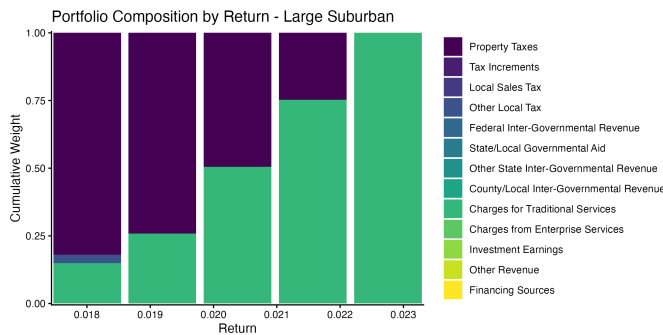
**Figure 10:** Large and Suburban Municipalities Comparing Symmetrical and Asymmetrical Efficient Portfolio Frontiers and Compositions



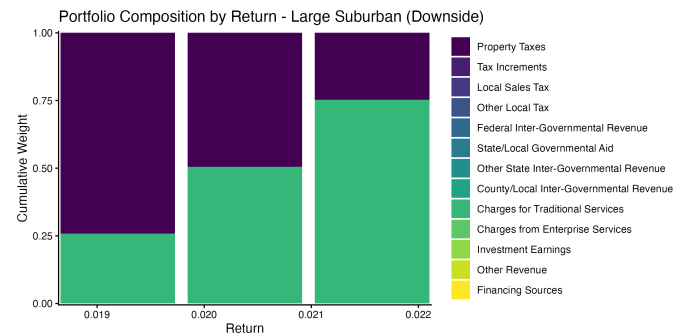
**(a)** Large and Suburban Municipalities Efficient Frontier



**(b)** Large and Suburban Municipalities Efficient Frontier - Downside



**(c)** Large and Suburban Municipalities Portfolio Composition



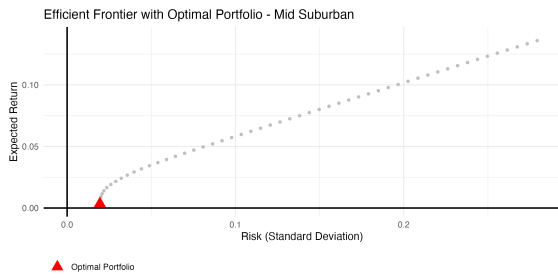
**(d)** Large and Suburban Municipalities Portfolio Composition - Downside

**Alt Text:** The Large suburban category efficient frontiers and portfolio composition in Figure 10 are very similar, with portfolio returns just below 2% and compositions of property taxes and charges for traditional services. The symmetric set of portfolios includes two options at 1.8% and 2.3% that are more concentrated in the sources of property taxes and charges for traditional services, respectively.

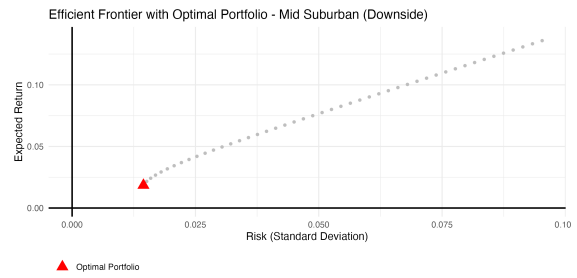
**6.4.2 Mid-Size Suburban Municipalities—** Property taxes and local sales tax constitute the majority of both symmetric and asymmetric efficient portfolios for mid-size suburban municipalities, with a small amount of charges for traditional services included in the lowest return-efficient portfolios (Figure 11, panels **c** and **d**). This differs from our large suburban category, where sales taxes are largely absent. The lowest-return revenue structures in the symmetric portfolio also include TIFs, charges for enterprise services, IGR, and other revenue sources. Asymmetric portfolios offer higher returns across the efficient frontier and exclude revenue sources such as IGR, TIFs, and charges for enterprise services and financing (Figure 11, panels **a** and **b**).

The presence of property taxes is consistent with our mid-size suburban case, Edina, at the lowest levels of return, although these are replaced more quickly and more completely in Edina's efficient portfolios than in the average mid-size suburban portfolio. Edina's frontier portfolios were also more diverse in composition and offered less growth than the average efficient portfolios for municipalities of its type, while neither of its symmetric or asymmetric portfolios included any sales taxes. This is not unexpected given the preponderance of residential zoning in Edina ([City of Edina Planning Department, 2015](#)).

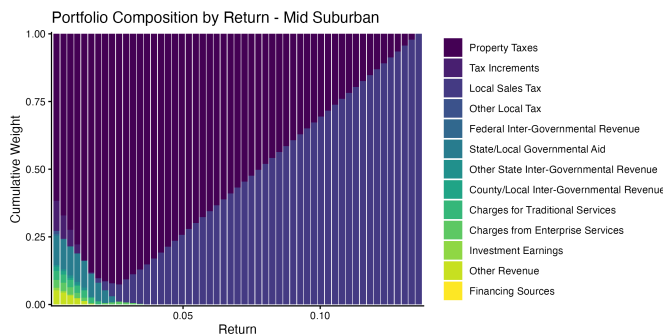
**Figure 11:** *Mid-Size and Suburban Municipalities Comparing Symmetrical and Asymmetrical Efficient Portfolio Frontiers and Compositions*



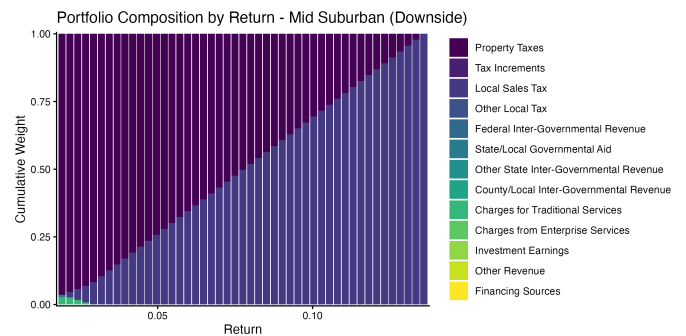
**(a)** *Mid-Size and Suburban Municipalities Efficient Frontier*



**(b)** *Mid-Size and Suburban Municipalities Efficient Frontier - Downside*



**(c)** *Mid-Size and Suburban Municipalities Portfolio Composition*

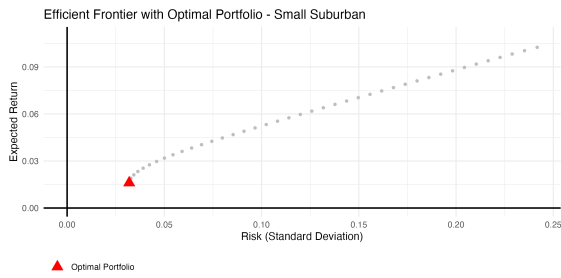


**(d)** *Mid-Size and Suburban Municipalities Portfolio Composition - Downside*

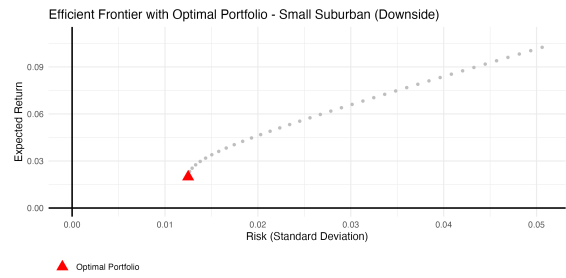
**Alt Text:** Figure 11 displays the mid-size suburban municipality category portfolios, including their frontiers and compositions. These are both comprised of property taxes and local sales taxes, with property taxes generally replaced as return rises. The symmetric portfolios include very low return structures, which include TIF revenue, multiple types of IGR, charges for enterprise services, investment earnings, and other revenue, that are not present in the asymmetric set.

**6.4.3 Small Suburban Municipalities**—Our small suburban municipalities have portfolio compositions much like their larger peers, as seen in (Figure 12, panels c and d), with property taxes replaced with sales taxes as return increases. Their efficient, symmetric portfolios also create less growth (Figure 12, panels a and b). The average efficient structure in this group differs substantially from that of Lake Elmo, which, with limited commercial zoning (?), has efficient portfolios without local sales tax. The differences between our cases and municipal categories suggest that the service profile of municipalities shapes their revenue structures, and, therefore, highlight the context required when considering the appropriate revenue strategy for a municipality, as we discuss further below.

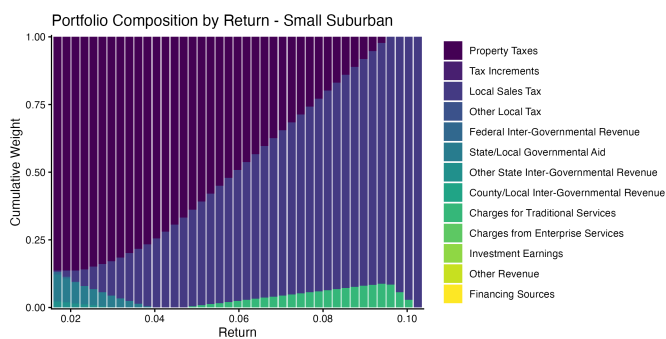
**Figure 12: Small and Suburban Municipalities Comparing Symmetrical and Asymmetrical Efficient Portfolio Frontiers and Compositions**



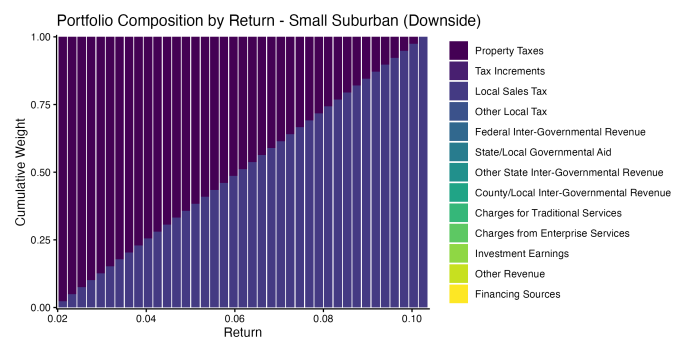
**(a) Small and Suburban Municipalities Efficient Frontier**



**(b) Small and Suburban Municipalities Efficient Frontier - Downside**



**(c) Small and Suburban Municipalities Portfolio Composition**

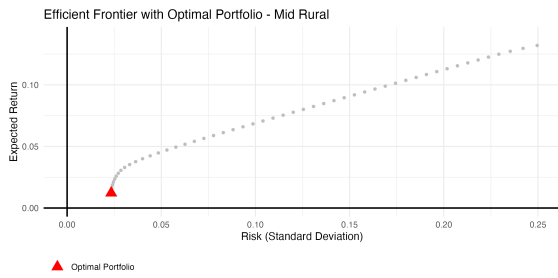


**(d) Small and Suburban Municipalities Portfolio Composition - Downside**

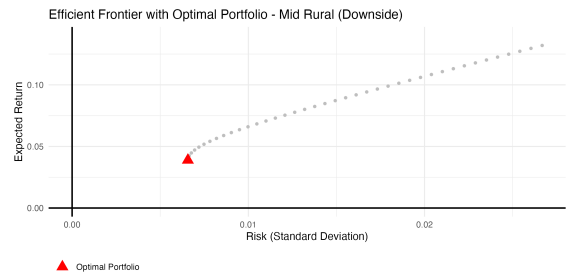
**Alt Text:** Figure 12 displays the frontiers and compositions of the small suburban category. These are similar to their mid-size peers in that property taxes and other local taxes are the majority of all efficient portfolios. The efficient frontiers for symmetric portfolios are of lower return at their lowest level of risk when compared to the asymmetric low-risk portfolio, at 1.6% as compared to 1.99%, respectively, and both reach a maximum return of about 10.2%.

**6.4.4 Midsize Rural Municipalities**—Midsize rural municipalities have efficient portfolio compositions that are similar to their midsize and small suburban peers, primarily composed of property taxes and local sales tax, although some intergovernmental revenue is present in the majority of the symmetrical frontier.

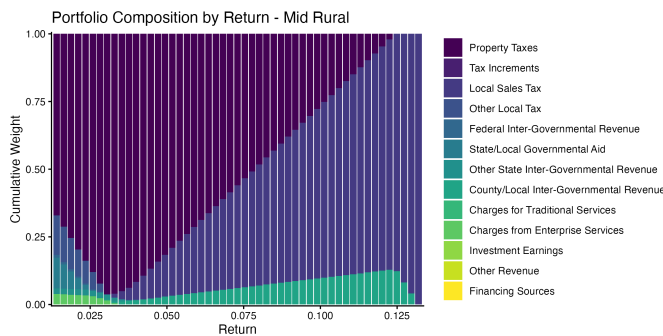
**Figure 13:** Mid-Size and Rural Municipalities Comparing Symmetrical and Asymmetrical Efficient Portfolio Frontiers and Compositions



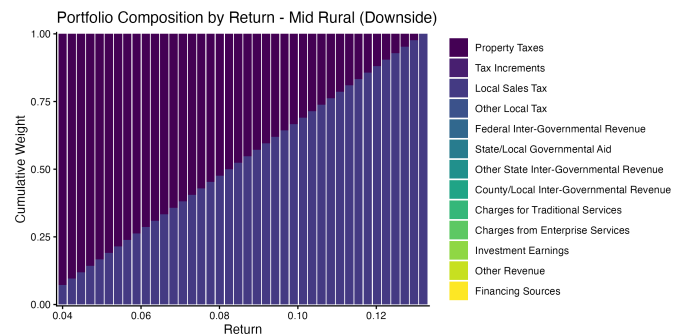
**(a)** Mid-Size and Rural Municipalities Efficient Frontier



**(b)** Mid-Size and Rural Municipalities Efficient Frontier - Downside



**(c)** Mid-Size and Rural Municipalities Portfolio Composition



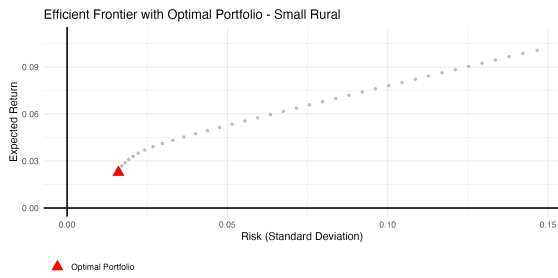
**(d)** Mid-Size and Rural Municipalities Portfolio Composition - Downside

**Alt Text:** The mid-size rural efficient frontiers and portfolio compositions of Figure 13 both include property taxes and local sales tax, similar to the mid-size and small suburban categories.

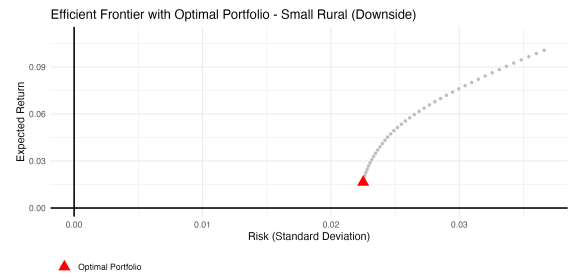
**6.4.5 Small Rural Municipalities**—The symmetric and asymmetric portfolios of small rural municipalities demonstrate the greatest differences of any of our categories in that property taxes are absent from the minimal risk downside-only portfolio, while charges for traditional services are absent from the minimal risk symmetric portfolio, even though they are the modal revenue in the majority of the efficient downside-only portfolios.

**Alt Text:** The small rural efficient frontiers and portfolio compositions of Figure 14 are quite different, with the symmetric portfolio structure very similar to the composition of the mid-size category, but the asymmetric set of portfolios relying on larger proportions of charges for enterprise services and state and local government aid while entirely excluding property taxes.

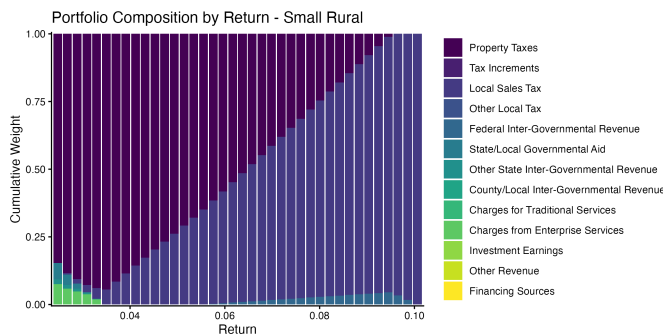
**Figure 14: Small and Rural Municipalities Comparing Symmetrical and Asymmetrical Efficient Portfolio Frontiers and Compositions**



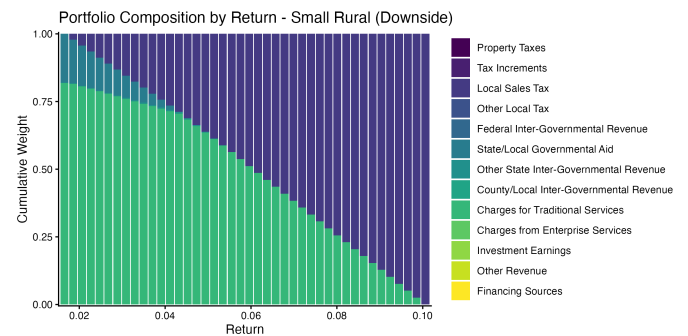
**(a) Small and Rural Municipalities Efficient Frontier**



**(b) Small and Rural Municipalities Efficient Frontier - Downside**



**(c) Small and Rural Municipalities Portfolio Composition**

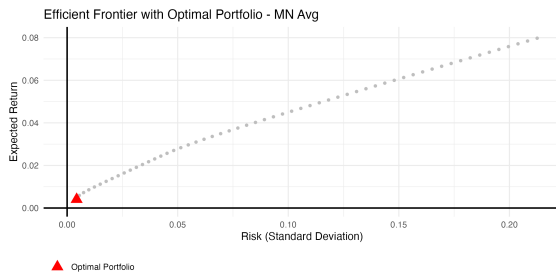


**(d) Small and Rural Municipalities Portfolio Composition - Downside**

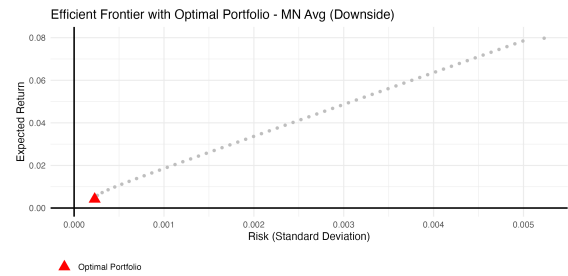
## 6.5 All Minnesota Municipalities, Average Portfolio

Examining the composition of the average efficient portfolios for municipalities in Minnesota, the results suggest the potential importance of asymmetric risk. The presence of local sales tax and property tax in the portfolios' structures is not surprising, but there are striking differences between the symmetric and asymmetric portfolios in both their composition (Figure 15, panels **c** and **d**) and risk (Figure 15, panels **a** and **b**). For example, sales tax is completely replaced in the symmetric portfolio as it approaches a 3% return, whereas it remains in almost all asymmetric portfolios (up to nearly 8% growth, the maximum). State and local government aid is a larger proportion of the symmetric portfolios. Property taxes constitute a larger share of symmetric portfolios, reflecting their stability but also indicating that this stability results from reduced growth. Small proportions of charges for services (traditional and enterprise) are present in the highest-return asymmetric portfolios.

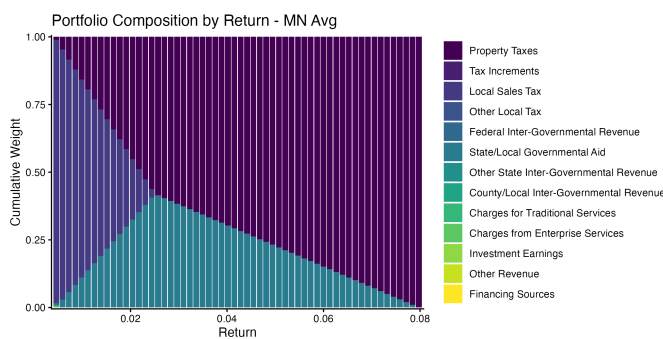
**Figure 15:** All Minnesota Municipalities Comparing Symmetrical and Asymmetrical Efficient Portfolio Frontiers and Compositions



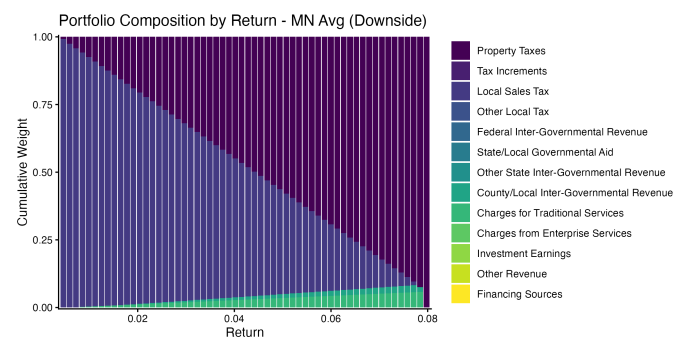
(a) All Minnesota Municipalities Efficient Frontier



(b) All Minnesota Municipalities Efficient Frontier - Downside



(c) All Minnesota Municipalities Portfolio Composition



(d) All Minnesota Municipalities Portfolio Composition - Downside

**Alt Text:** Figure 15 includes the efficient frontiers and compositions calculated using data from all Minnesota municipalities. These range from a return of about 0.4% for the lowest-risk portfolios to just above 0.8% growth. The symmetric set of portfolios includes much more state and local government aid, while both portfolios rely on property taxes at maximum return.

## 7 Discussion

### 7.1 Implications of PMPT for Public Administration

Although post-modern portfolio approaches may seem abstract and theoretical, they have clear implications for major issues in public finance and financial management. This includes two major issues: the use of HHI as a measure and the reliance on symmetrical measures of risk. We understand each of these when we examine the two perspectives on municipal revenue diversification identified by (Jimenez and Afonso, 2022): financial management and fiscal illusion. The fact that these issues have implications for both perspectives is quite powerful, as they view diversification differently: the first regards diversification as a

potential buffer against exogenous shocks, whereas the second regards diversification as a potential barrier to either transparency or the evaluation of the true cost of government, as taxpayers cannot effectively monitor complex revenue structures or may discount the tax burden from particular streams.

Examining the first issue, HHI, we argue that using a traditional measure of diversification would pose problems for a quantitative study focused on either topic, as the HHI index would not provide sufficient information to predict the dependent variables used to measure financial health or tax effort. Without information on the composition of a revenue structure, one might expect a structure with a specific composition that is more or less diverse than average to associate with varied outcomes of either. For example, a diverse set of revenues that are positively correlated and pro-cyclical may increase vulnerability, whereas negative associations among sources of varying cyclicity may provide effective buffering. If applied to the efficiency of capital projects, as suggested by Boasson, Cheng, and Boasson (2012), the risk and return of each project would affect overall performance, and the benefits of portfolio diversification would depend on the performance of each asset and the nature of their covariation. Similarly, for fiscal illusion, a diverse set of elastic revenues may lead to unconstrained growth, whereas a diverse set of inelastic revenues may constrain it. So, in each of these cases, only specific information from the portfolio can provide greater certainty. By incorporating this information, modern portfolio theory can better address these questions.

Despite this advantage of MPT over HHI, it remains limited by its treatment of risk, highlighting the second major issue for public administration scholarship that upside and downside risk are unlikely to be of equal importance. Given this, post-modern approaches utilize sufficient information to address questions in both areas more effectively. In financial management, covariances between revenues or assets are critical to discussions of diversification, as inverse associations provide the fundamental rationale for diversification. Modern portfolio theory addresses this, but its risk calculations are misleading when organizations' upside and downside risk tolerances differ. In these cases, post-modern approaches that account for asymmetric risk preferences will provide a more meaningful tool for municipal decision-makers. In the case of fiscal illusion, the ability to examine individual streams, their growth, and compare their upside and downside variances to economic conditions provides the capacity to identify those that fill financial gaps when other revenues decline rather than increasing the tax burden, while also identifying those that are likely to be elastic and possibly associated with increased tax effort or ratcheting.

## **7.2 Implications of Results for Municipal Financial Management Practice**

The results of our revenue optimizations have implications for municipal financial management in practice. The first of these is a note of caution about revenue diversification. The portfolios identified as optimal in

this study are concentrated; those that minimize downside risk are particularly so. Many portfolios that minimize downside risk are highly dependent on a single revenue source, with the modal revenue source exceeding 50% of the overall revenue structure. Among symmetric portfolios, only Minneapolis, St. Paul, Duluth, and St. Cloud lack a modal revenue category that exceeds 50%, and none of the portfolios constructed for municipality groups have a modal category below 50%. Although there are many limitations that should condition the application of these results, as discussed below, they do suggest that even the most commonly expressed rationale for diversification, risk reduction, would be unlikely to be realized via diversification among Minnesota municipalities. This has direct applicability to revenue management practice; revenue strategies, including diversification, should be evaluated in the context of the municipality, and absent this, expectations for the applicability of scholarship to practice should be tempered.

The results also suggest that the tolerance for upside risk is likely to be important for revenue management. Although the specific structures of the portfolios are not intended to serve as a set of practical ideals, the differences between symmetrical and asymmetrical portfolios are often stark, with downside-only optimizations consistently more concentrated in our examples, and we note that they also include or exclude revenue streams relative to the symmetric revenue structures. While the tradeoffs between growth and ratcheting are beyond the scope of this analysis, it is reasonable to expect concerns about ratcheting, or other factors related to upside risk, to change portfolio preferences.

Finally, there are major differences between our cases, as well as between these cases and the categories of municipalities of which they are members. This method cannot explain those differences, but the results suggest that revenue management is highly contextual.

### **7.3 Research Agenda**

We believe that asymmetric measures and the application of post-modern portfolio theory have many potential applications within public administration and public policy. Initially, further work could focus on improving the applicability of scholarship to the practice of revenue management. This paper only focuses on theoretically optimal portfolios; it imposes almost no constraints on the proportion of each portfolio contributed by a revenue source, so future work could set bounds based on what has been historically possible, impose incremental limits on change based on prior composition, or constrain values by what is feasible given a particular revenue source by incorporating information from tax expenditure limits, state revenue sharing formulas, or contextual variables. Related, exogenous events, such as state policy, could influence both revenues and the desirability of revenue structures, so identifying theoretically optimal portfolios before and after these events might inform policy.

Studies could also examine elements of revenue structures. For example, the efficiency of tax structures could be examined using information on the composition of the tax base and the tax revenues generated from it, which might have implications for zoning preferences or tax decisions within the discretion of local authorities.

The efficiency of revenue structures may also have implications for assessing municipal risk, so associations between revenue-structure efficiency and municipal bond ratings may be relevant to scholarship and practice. The inclusion of asymmetric risk, focusing on downside variance, would be particularly critical for these studies.

The capacity to partition risk into its upside and downside would also convey advantages for the study of budget ratcheting due to fiscal illusions, as upside variance could be associated with changes in future expenditures in order to identify ratcheting effects.

## 8 Conclusion

This study demonstrates the importance of asymmetry in risk measurement; highlights the importance of context, suggests that a traditional measure of revenue structures, HHI, will be problematic for both scholarship and practice; and demonstrates the potential value of optimization techniques for the study of municipal revenue structures.

The consistent differences between our symmetrical and asymmetric portfolios demonstrate that the weight assigned to upside or downside risk will affect what is considered desirable. Given that some discounting of upside risk likely reflects the reality of municipal governments, variance-based risk measurement will benefit from partitioning risk. The weighted risk measure deployed here provides a mechanism to capture heterogeneous preferences while accounting for both variance and covariance.

Examining both select cases and types of municipalities in Minnesota underscores the importance of context for municipal governments. The results indicate portfolios with varying diversity, with some municipalities' optimal portfolios substantially more concentrated or more diverse than others along the efficient frontier. Although we caution against using these results for practical revenue management, the selection of a revenue diversification strategy would be municipality-specific and dependent on factors such as current composition and risk tolerance.

It also demonstrates why measures that lack granularity, which can be understood relative to this context, such as HHI, will fail to provide useful information. The optimal portfolios identified above, particularly those focused on minimizing downside risk, are relatively concentrated and composed of very specific revenue sources.

Of course, this study also has limitations. While its results are suggestive, they should not be confused with attainable or desirable revenue structures in these communities. Models do not incorporate information on the municipal characteristics that facilitate or limit revenue capture, boundaries created by policy or other factors that may constrain growth, or time-specific information about changes to state revenue-sharing models. Instead, they represent theoretical efficiency over a given period. Implementing analysis with such characteristics or constraints is possible if it is focused on an individual case, but the conceptual framework required for this type of modeling is undeveloped and beyond the scope of this study.

The efficiency of individual revenue streams is also assumed away in these models, which cannot capture the costs of the expenses associated with revenue streams. Instead, optimizing for revenue growth assumes that they are not generating net losses, a major assumption that we cannot test with available data.

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# Appendix

## A `PMPOptimizer` R Package

Written alongside this paper, an R package, `PMPOptimizer`, was developed to generate optimal portfolios and efficient frontiers using weighted covariance matrices, along with their corresponding visualizations. The development version of this package is available on a [GitHub repository](#), and we anticipate submitting the package to the CRAN repository in the future. An analogous Python package is also planned and will be maintained in the same repository as the R package.

Although the package was developed in support of this paper, we hope it will have broader applications. Any setting involving constrained optimization in which a risk-aversion parameter could play a role may benefit from the functionality provided by this R package, potentially yielding useful or otherwise interesting insights.

## B Optimal Portfolio Composition

The tables below reflect the optimal portfolio compositions for both the symmetric and downside-weighted computations.

**Table A1:** *Optimal Portfolio Weighting—Minneapolis*

Category	Symmetric	Downside
Property Taxes	0.1583	0.7595
Tax Increments	-	-
Local Sales Tax	0.0391	-
Other Local Tax	0.0963	0.1058
Federal Inter-Governmental Revenue	0.0595	-
State/Local Governmental Aid	0.3154	0.0597
Other State Inter-Governmental Revenue	0.0481	-
County/Local Inter-Governmental Revenue	0.0086	-
Charges for Traditional Services	0.0855	0.075
Charges from Enterprise Services	0.063	-
Investment Earnings	0.0225	-
Other Revenue	0.0535	-
Financing Sources	0.0502	-
Expected Return	-0.6%	2.98%
Minimized SD	0.0345	0.0451

**Table A2: Optimal Portfolio Weighting–St. Paul**

Category	Symmetric	Downside
Property Taxes	0.2673	0.9099
Tax Increments	-	-
Local Sales Tax	0.2902	0.09
Other Local Tax	-	-
Federal Inter-Governmental Revenue	0.0446	-
State/Local Governmental Aid	0.2214	-
Other State Inter-Governmental Revenue	-	-
County/Local Inter-Governmental Revenue	-	-
Charges for Traditional Services	0.1568	-
Charges from Enterprise Services	0.0095	-
Investment Earnings	0.0101	-
Other Revenue	-	-
Financing Sources	-	-
Expected Return	0.621%	3.27%
Minimized SD	0.0214	0.0104

**Table A3: Optimal Portfolio Weighting–Duluth**

Category	Symmetric	Downside
Property Taxes	0.2119	-
Tax Increments	-	-
Local Sales Tax	0.0932	0.9295
Other Local Tax	0.1022	0.0006
Federal Inter-Governmental Revenue	-	-
State/Local Governmental Aid	0.486	0.0699
Other State Inter-Governmental Revenue	0.0206	-
County/Local Inter-Governmental Revenue	0.0169	-
Charges for Traditional Services	0.0325	-
Charges from Enterprise Services	-	-
Investment Earnings	-	-
Other Revenue	0.0367	-
Financing Sources	-	-
Expected Return	0.651%	2.82%
Minimized SD	0.0219	0.0113

**Table A4: Optimal Portfolio Weighting–Rochester**

Category	Symmetric	Downside
Property Taxes	0.7041	0.9849
Tax Increments	0.0536	-
Local Sales Tax	-	0.013
Other Local Tax	-	-
Federal Inter-Governmental Revenue	-	-
State/Local Governmental Aid	0.0657	-
Other State Inter-Governmental Revenue	-	-
County/Local Inter-Governmental Revenue	0.0113	-
Charges for Traditional Services	0.1495	0.002
Charges from Enterprise Services	-	-
Investment Earnings	0.0157	-
Other Revenue	-	-
Financing Sources	-	-
Expected Return	3.38%	4.77%
Minimized SD	0.0212	0.0029

**Table A5: Optimal Portfolio Weighting–Edina**

Category	Symmetric	Downside
Property Taxes	0.7967	0.9918
Tax Increments	0.0146	-
Local Sales Tax	-	-
Other Local Tax	0.0075	-
Federal Inter-Governmental Revenue	0.0009	-
State/Local Governmental Aid	-	-
Other State Inter-Governmental Revenue	0.0054	-
County/Local Inter-Governmental Revenue	-	-
Charges for Traditional Services	0.1418	0.0073
Charges from Enterprise Services	0.0331	0.001
Investment Earnings	-	-
Other Revenue	-	-
Financing Sources	-	-
Expected Return	2.42%	2.56%
Minimized SD	0.0224	0.0083

**Table A6: Optimal Portfolio Weighting–Lake Elmo**

Category	Symmetric	Downside
Property Taxes	0.7286	0.9979
Tax Increments	-	-
Local Sales Tax	-	-
Other Local Tax	0.1007	0.0006
Federal Inter-Governmental Revenue	-	-
State/Local Governmental Aid	-	-
Other State Inter-Governmental Revenue	0.0201	-
County/Local Inter-Governmental Revenue	-	-
Charges for Traditional Services	0.0079	-
Charges from Enterprise Services	0.0152	0.0001
Investment Earnings	0.1027	0.0014
Other Revenue	-	-
Financing Sources	0.0248	-
Expected Return	4.31%	5.77%
Minimized SD	0.0558	0.0189

**Table A7: Optimal Portfolio Weighting–St. Cloud**

Category	Symmetric	Downside
Property Taxes	0.3357	0.1708
Tax Increments	0.0073	-
Local Sales Tax	0.1405	-
Other Local Tax	0.174	0.2003
Federal Inter-Governmental Revenue	0.0421	-
State/Local Governmental Aid	0.2137	0.6288
Other State Inter-Governmental Revenue	-	-
County/Local Inter-Governmental Revenue	0.0092	-
Charges for Traditional Services	-	-
Charges from Enterprise Services	0.0775	-
Investment Earnings	-	-
Other Revenue	-	-
Financing Sources	-	-
Expected Return	-1.00%	-0.88%
Minimized SD	0.0286	0.0462

**Table A8: Optimal Portfolio Weighting–Bemidji**

Category	Symmetric	Downside
Property Taxes	0.2061	0.102
Tax Increments	0.0216	-
Local Sales Tax	0.0235	0.0329
Other Local Tax	0.1306	0.0417
Federal Inter-Governmental Revenue	-	-
State/Local Governmental Aid	0.5484	0.8234
Other State Inter-Governmental Revenue	-	-
County/Local Inter-Governmental Revenue	0.0501	-
Charges for Traditional Services	0.0198	-
Charges from Enterprise Services	-	-
Investment Earnings	-	-
Other Revenue	-	-
Financing Sources	-	-
Expected Return	0.07%	-0.97%
Minimized SD	0.0254	0.0558

**Table A9: Optimal Portfolio Weighting–Wadena**

Category	Symmetric	Downside
Property Taxes	0.3355	0.0676
Tax Increments	0.0086	-
Local Sales Tax	-	-
Other Local Tax	-	-
Federal Inter-Governmental Revenue	-	-
State/Local Governmental Aid	0.6091	0.9324
Other State Inter-Governmental Revenue	-	-
County/Local Inter-Governmental Revenue	-	-
Charges for Traditional Services	0.0434	-
Charges from Enterprise Services	-	-
Investment Earnings	-	-
Other Revenue	0.0034	-
Financing Sources	-	-
Expected Return	0.92%	1.18%
Minimized SD	0.0441	0.0266

**Table A10: Optimal Portfolio Weighting—Large and Suburban**

Category	Symmetric	Downside
Property Taxes	0.8122	0.9142
Tax Increments	-	-
Local Sales Tax	-	-
Other Local Tax	0.0503	-
Federal Inter-Governmental Revenue	0.001	-
State/Local Governmental Aid	-	-
Other State Inter-Governmental Revenue	0.0056	-
County/Local Inter-Governmental Revenue	-	-
Charges for Traditional Services	0.1279	0.0858
Charges from Enterprise Services	0.003	-
Investment Earnings	-	-
Other Revenue	-	-
Financing Sources	-	-
Expected Return	1.7%	1.84%
Minimized SD	.022	0.0123

**Table A11: Optimal Portfolio Weighting—Mid-Size and Suburban**

Category	Symmetric	Downside
Property Taxes	0.6117	0.9527
Tax Increments	0.1134	-
Local Sales Tax	-	-
Other Local Tax	0.0149	-
Federal Inter-Governmental Revenue	0.0059	-
State/Local Governmental Aid	0.1079	0.0284
Other State Inter-Governmental Revenue	0.0068	-
County/Local Inter-Governmental Revenue	0.0152	-
Charges for Traditional Services	0.0339	0.0189
Charges from Enterprise Services	0.0277	-
Investment Earnings	0.0091	-
Other Revenue	0.0534	-
Financing Sources	-	-
Expected Return	0.36%	1.85%
Minimized SD	0.0195	0.0145

**Table A12: Optimal Portfolio Weighting–Small and Suburban**

Category	Symmetric	Downside
Property Taxes	0.8616	0.9914
Tax Increments	-	-
Local Sales Tax	-	0.0086
Other Local Tax	-	-
Federal Inter-Governmental Revenue	0.0102	-
State/Local Governmental Aid	0.1039	-
Other State Inter-Governmental Revenue	0.0243	-
County/Local Inter-Governmental Revenue	-	-
Charges for Traditional Services	-	-
Charges from Enterprise Services	-	-
Investment Earnings	-	-
Other Revenue	-	-
Financing Sources	-	-
Expected Return	1.61%	1.99%
Minimized SD	0.0319	0.0125

**Table A13: Optimal Portfolio Weighting–Mid-Size and Rural**

Category	Symmetric	Downside
Property Taxes	0.6372	0.9392
Tax Increments	-	-
Local Sales Tax	-	0.0604
Other Local Tax	0.158	-
Federal Inter-Governmental Revenue	0.0108	-
State/Local Governmental Aid	0.133	-
Other State Inter-Governmental Revenue	0.0209	-
County/Local Inter-Governmental Revenue	0.0003	0.0004
Charges for Traditional Services	-	-
Charges from Enterprise Services	0.0398	-
Investment Earnings	-	-
Other Revenue	-	-
Financing Sources	-	-
Expected Return	1.22%	3.89%
Minimized SD	0.0234	0.0066

**Table A14: Optimal Portfolio Weighting–Small and Rural**

Category	Symmetric	Downside
Property Taxes	0.7964	-
Tax Increments	-	-
Local Sales Tax	-	-
Other Local Tax	-	-
Federal Inter-Governmental Revenue	-	-
State/Local Governmental Aid	0.0863	0.182
Other State Inter-Governmental Revenue	0.0226	0.0021
County/Local Inter-Governmental Revenue	-	-
Charges for Traditional Services	-	0.8159
Charges from Enterprise Services	0.0946	-
Investment Earnings	-	-
Other Revenue	-	-
Financing Sources	-	-
Expected Return	2.27%	1.65%
Minimized SD	0.016	0.0225

**Table A15: Optimal Portfolio Weighting–State of Minnesota**

Category	Symmetric	Downside
Property Taxes	-	0.0019
Tax Increments	-	-
Local Sales Tax	0.9721	0.9981
Other Local Tax	-	-
Federal Inter-Governmental Revenue	-	-
State/Local Governmental Aid	0.011	-
Other State Inter-Governmental Revenue	-	-
County/Local Inter-Governmental Revenue	-	-
Charges for Traditional Services	-	-
Charges from Enterprise Services	0.0168	-
Investment Earnings	-	-
Other Revenue	-	-
Financing Sources	-	-
Expected Return	0.41%	0.42%
Minimized SD	0.0043	0.0002